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Study of Mean Time to Lose Lock and Lock Detector Threshold in GPS Carrier Tracking Loops*

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Abstract — Narrow-Band power detector was applied to indicate the lock status in tracking loops of GPS receiver. Previous researches did not discover the statistical characteristics and mean time to lose lock of detector, thus the threshold selection did not have theoretical support. In this paper, the probability distribution of detector is derived theoretically by stochastic method, and the relationship between the lock probability and carrier to noise (C/N_0) of the signal is discovered. Furthermore, the paper analyzes the Mean time to lose lock (MTLL) related to threshold setting and C/N_0 of signal, providing a theoretical basis for threshold setting method of GPS tracking loops. At last, the simulations and real data test have been made to prove the results.

Key words — Mean time to lose lock (MTLL), Lock probability, GPS receiver, Phase locked loop.

I. Introduction

In Global Positioning System (GPS) receivers, lock detection mechanism was used to indicate signal tracking status by comparing with a threshold. Though the signals tracking in code, frequency and phase have different indicators, the phase lock was more commonly used as other locks[1]. The Phase lock detector (PLD) was adopted as indicator because Phase locked loop (PLL) was more sensitive than Frequency locked loop (FLL) and Delay locked loop (DLL).

The phase locked loop/delay locked loop was developed to track GPS signals. Spilker[1] provided a method using the normalized estimate of the cosine of twice the carrier phase to calculate the lock but none of statistic distribution characteristics of the lock has been analyzed. In his book, the threshold just set to 0.4 without any theoretical support. Mileant and Hinedi[2] gave the square law and absolute value type detectors used to quantify the function between lock probability and carrier to noise power (C/N_0) under a specific false alarm probability in phase locked loop. Linn and Peleg[3] suggested a family of carrier lock detectors for M-PSK receivers operating in additive white Gaussian noise channels, with the statistical properties of lock detector derived theoretically.

In the paper, Section II presents an overview of the general structure of the received GPS signal. Section III introduces the phase lock detector structure of GPS carrier tracking loop. Section IV analyzes the statistical characteristics and the Probability distribution function (PDF) of PLD. The approximate calculation methods are studied under different C/N_0 with simulation to prove them. Section V further outlines the relationship of detection probability and continuous tracking performance of PLD to determinate optimal detection threshold setting. The Section VI makes conclusions and describes the future work. The real GPS data sampled at March 2011 is used to prove the result. The data length is 970 seconds which contains 8 satellites with C/N_0 at $40 \sim 50$dBHz.

II. Signal Modeling

In GPS receivers, received signals from antenna are demodulated to intermediate frequency through low noise amplifiers, filters and down-converters. The intermediate frequency signal $S(t)$ is shown in Eq.(1).

$$S(t) = AC(t)D(t)\cos[(w_{IF} + w_{doppler})t + \phi_0] + n(t) \tag{1}$$

where A is the received carrier amplitude, $C(t)$ is the GPS C/A PRN, $D(t)$ is the navigation data, w_{IF} is the intermediate frequency of GPS signals, $w_{doppler}$ is the Doppler frequency for relative motion between satellites and receivers, ϕ_0 is the carrier accumulated phase, $n(t)$ is a Gaussian white noise (GWN) following normal distribution.

Through the front-end A/D sampling, quantization and accumulation, the in- and quadrature-phase accumulations are shown in Eqs.(2) and (3).

$$I_i = \sqrt{\frac{2ST}{N_0}} R(\Delta\tau)\sin c(\pi\Delta f_i T)\cos(\pi\Delta f_i T + \Delta\theta_i) + n_{I,i} \tag{2}$$

$$Q_i = \sqrt{\frac{2ST}{N_0}} R(\Delta\tau)\sin c(\pi\Delta f_i T)\sin(\pi\Delta f_i T + \Delta\theta_i) + n_{Q,i} \tag{3}$$

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where S/N_0 is the signal to noise density in dB-Hz, T is the time length of accumulation, $R(\Delta\tau)$ is the self-correlation function of PRN code, $\Delta\tau$ is the code phase error between received and local replica, Δf_i is the frequency error between received and local replica, $\Delta\theta$ is the phase error, $n_{I,i}$ and $n_{Q,i}$ are standard GWN (independent and identically distributed denoted by $n_{I,i} \sim N(0.1)$, $n_{Q,i} \sim N(0,1)$).

III. Structure of Lock Detectors

Phase lock detector in traditional carrier tracking loop is achieved through narrow-band detector normalized by narrow-band signal power over a period. The Equation of the detector is Eq.(4), which have been described in Ref.[1].

$$C_{2\varphi} = NBD_k/NBP_k \tag{4}$$

where

$$NBD_k = \left(\textstyle\sum_{i=1}^{M} I_i\right)^2 - \left(\textstyle\sum_{i=1}^{M} Q_i\right)^2,$$
$$NBP_k = \left(\textstyle\sum_{i=1}^{M} I_i\right)^2 + \left(\textstyle\sum_{i=1}^{M} Q_i\right)^2,$$

M is the times of accumulation.

After mapping from Cartesian coordinates to a polar system, De Moivre's theorem[4] is used to simplify Eq.(4).

$$\begin{aligned} NBD_k =& \left(\textstyle\sum_{i=1}^{M} I_i\right)^2 - \left(\textstyle\sum_{i=1}^{M} Q_i\right)^2 \\ =& \mathrm{Re}\left\{\left(\textstyle\sum_{i=1}^{M} I_i + j\textstyle\sum_{i=1}^{M} Q_i\right)^2\right\} \\ =& \left[\left(\textstyle\sum_{i=1}^{M} I_i\right)^2 + \left(\textstyle\sum_{i=1}^{M} Q_i\right)^2\right]\mathrm{Re}\{[\cos\varphi + j\sin(\varphi)]^2\} \\ =& \left[\left(\textstyle\sum_{i=1}^{M} I_i\right)^2 + \left(\textstyle\sum_{i=1}^{M} Q_i\right)^2\right]\cos(2\varphi) \end{aligned} \tag{5}$$

$$\begin{aligned} C_{2\varphi} =& \frac{NBD_k}{NBP_k} \\ =& \frac{\left(\sum_{i=1}^{M} I_i\right)^2 - \left(\sum_{i=1}^{M} Q_i\right)^2}{\left(\sum_{i=1}^{M} I_i\right)^2 + \left(\sum_{i=1}^{M} Q_i\right)^2} \\ =& \frac{\left[\left(\sum_{i=1}^{M} I_i\right)^2 + \left(\sum_{i=1}^{M} Q_i\right)^2\right]\cos(2\varphi)}{\left(\sum_{i=1}^{M} I_i\right)^2 + \left(\sum_{i=1}^{M} Q_i\right)^2} \\ =& \cos(2\varphi) \end{aligned} \tag{6}$$

where φ is the average phase error over $M \times T$ (sec). When in lock, $R(\Delta\tau) \approx R(0) = 1$ and $\Delta f_i = 0$. Based on Ref.[5], the relationship between φ and phase in correlator $\Delta\theta$ can be derived by

$$\begin{aligned} \varphi =& ar\tan\left(\frac{\sum_{i=1}^{M} Q_i}{\sum_{i=1}^{M} I_i}\right) \\ =& ar\tan\left(\frac{\sqrt{\frac{2ST}{N_0}}\sin(\Delta\bar{\theta}) + \frac{1}{M}\sum_{i=1}^{M} n_{Q,i}}{\sqrt{\frac{2ST}{N_0}}\cos(\Delta\bar{\theta}) + \frac{1}{M}\sum_{i=1}^{M} n_{I,i}}\right) \end{aligned} \tag{7}$$

Here we define $n_I(M)$ and $n_Q(M)$ as the summary noise during accumulation time M. Thus,

$$n_I(M) = \frac{1}{M}\sum_{i=1}^{M} n_{I,i}, n_Q(M) = \frac{1}{M}\sum_{i=1}^{M} n_{Q,i} \tag{8}$$

The mathematical expectation and variance of them are shown in Eqs.(9) and (10).

$$E\{n_Q(M)\} = E\{n_I(M)\} = 0 \tag{9}$$
$$VAR\{n_Q(M)\} = VAR\{n_I(M)\} = 1/M \tag{10}$$

When putting Eq.(8) into Eq.(7), we will get

$$\begin{aligned} \varphi =& ar\tan\left(\frac{\bar{Q} + n_Q(M)}{\bar{I} + n_I(M)}\right) \\ \approx& ar\tan\left(\frac{\bar{Q}}{\bar{I}}\right) + \frac{1}{1+x^2}\bigg|_{I/Q}\left(\frac{\bar{Q} + n_Q(M)}{\bar{I} + n_I(M)} - \frac{\bar{Q}}{\bar{I}}\right) \\ =& ar\tan\left(\frac{\bar{Q}}{\bar{I}}\right) + \frac{\bar{I}^2}{\bar{I}^2 + \bar{Q}^2}\left(\frac{\bar{I} \times n_Q(M) - \bar{Q} \times n_Q(M)}{\bar{I}(\bar{I} + n_I(M))}\right) \\ \approx& \Delta\bar{\theta} + \frac{\bar{I} \times n_Q(M) - \bar{Q} \times n_I(M)}{\bar{I}^2 + \bar{Q}^2} \end{aligned} \tag{11}$$

where $\bar{I} = \sqrt{\frac{2ST}{N_0}}\cos(\Delta\bar{\theta})$, $\bar{Q} = \sqrt{\frac{2ST}{N_0}}\sin(\Delta\bar{\theta})$.

Compared to the surplus in Eq.(11), we have the estimation error listed in Eq.(12).

$$E_\varphi(M) = \frac{\bar{I} \times n_Q(M) - \bar{Q} \times n_I(M)}{\bar{I}^2 + \bar{Q}^2} \tag{12}$$

Thus, the mean and variance of $E_\varphi(M)$ are,

$$\begin{aligned} E\{\varphi\} =& E\{\Delta\bar{\theta} + E_\varphi(M)\} \\ =& \frac{\bar{I} \times E\{n_{Q,M}\} - \bar{Q} \times E\{n_{I,M}\}}{\bar{I}^2 + \bar{Q}^2} \\ =& 0 \end{aligned} \tag{13}$$

$$\begin{aligned} VAR\{\varphi\} =& VAR\{\Delta\bar{\theta} + E_\varphi(M)\} \\ =& \frac{\bar{I}^2}{(\bar{I}^2 + \bar{Q}^2)^2} \times VAR\{n_Q(M)\} \\ &+ \frac{\bar{Q}^2}{(\bar{I}^2 + \bar{Q}^2)^2} \times VAR\{n_I(M)\} \\ =& 1/M(\bar{I}^2 + \bar{Q}^2) \\ =& 1/(2S/N_0) \end{aligned} \tag{14}$$

In practical, phase lock detector result inside one second could be averaged to get a smoother result, which means $M \times T = 1$.

IV. Distribution of Phase Lock Detector

The phase error φ has the distribution as the phase of a Rice random variable within $|\varphi| \leq \pi$. Its probability density function is shown in Eq.(15)[6].

$$\begin{aligned} p_\varphi(\varphi) =& \frac{1}{2\pi}e^{\frac{S}{N_0}} + \frac{1}{2\pi}\sqrt{\frac{2S}{N_0}}\cos(\varphi) \\ &\times e^{-\frac{S}{N_0}\sin^2(\varphi)}\int_{-\infty}^{\cos(\varphi)\sqrt{2S/N_0}} e^{-\frac{x^2}{2}}dx \end{aligned} \tag{15}$$

The nature of PLD represents a Gaussian random process transits a cosine system. The mathematical expectation and variance of the PLD are shown in Eqs.(16) and (17).

$$\begin{aligned} E\{C_{2\varphi}\} =& E\{\cos(2\varphi)\} \\ =& \int_{-\infty}^{\infty} \cos(2\varphi) p(\varphi) d\varphi \\ \approx& \sqrt{\frac{S}{N_0\pi}} \int_{-\pi}^{\pi} \cos(2\varphi) \exp\left(-\frac{S}{N}\varphi^2\right) d\varphi \\ =& \exp\left(-\frac{1}{S/N_0}\right) \end{aligned} \tag{16}$$

$$\begin{aligned} VAR\{C_{2\varphi}\} =& E\{C_{2\varphi}^2\} - E^2\{C_{2\varphi}\} \\ =& \sqrt{\tfrac{S}{N_0\pi}} \int_{-\pi}^{\pi} \cos^2(2\varphi) e^{-\frac{S}{N_0}\varphi^2} d\varphi - e^{-\frac{2}{S/N_0}} \\ =& \frac{1}{2}\left(1 + e^{-\frac{4}{S/N_0}}\right) - e^{-\frac{2}{S/N_0}} \end{aligned} \tag{17}$$

Due to the complex nature of $p_\varphi(\varphi)$, we cannot derive PD of $C_{2\varphi}$ directly. In order to workaround it, the approximate performances of PLD under low and high C/N_0 are used to make further analysis. Under high SNR circumstance, the phase error φ is nearly Gaussian distribution which follows $N(0, 1/(2S/N_0))$. And under low SNR circumstance, it follows uniform distribution like $\varphi \sim U(-\pi, \pi)$. Based on the distribution of φ under different SNR situation, we can analyze the distribution characteristics of PLD briefly.

1. PLD under high SNR

Approximate procedure is used under high SNR situation. Here, the signals are locked and phase error is stable near zero. The PLD could be simplified as

$$C_{2\varphi} = \cos(2\varphi) \approx 1 - 2\varphi^2 \tag{18}$$

Based on Eq.(14), we define $\sigma^2 = 1/(2S/N_0)$. Putting it into Eq.(18), we will have the random variable Y which follows the chi-square distribution,

$$Y = \frac{(1 - C_{2\varphi})}{2\sigma^2} = \left(\frac{\varphi}{\sigma}\right)^2 - \chi^2(1) \tag{19}$$

According to Eq.(19), SNR is one of the main factors in PLD distribution. And we define $p_{\chi^2}(y)$ indicates PDF of random variable $Y \sim \chi^2(1)$, we have the PDF of PLD denoted by $p_{C_{2\varphi}}(C_{2\varphi})$,

$$p_{C_{2\varphi}}(C_{2\varphi}) = \frac{1}{2\sigma^2} p_{\chi^2}\left(\frac{1 - C_{2\varphi}}{2\sigma^2}\right), \quad |C_{2\varphi}| \le 1 \tag{20}$$

2. PLD under low SNR

In low SNR environment, the PDF of φ could be simplified to Eq.(21).

$$p_\varphi(\varphi) = \frac{1}{2\pi}, \quad |\varphi| \le \pi \tag{21}$$

Thus, the PLD follows arcsine distribution in Eq.(22).

$$f_{C_{2\varphi}}(C_{2\varphi}) = \frac{1}{\pi\sqrt{(1 + C_{2\varphi})(1 - C_{2\varphi})}}, \quad |C_{2\varphi}| \le 1 \tag{22}$$

The mathematical expectation and variance of $C_{2\varphi}$ described in Eqs.(23) and (24).

$$E\{C_{2\varphi}\} = \int_{-\infty}^{+\infty} \cos(2\varphi) f(\varphi) d\varphi = 0 \tag{23}$$

$$\begin{aligned} VAR\{C_{2\varphi}\} =& E\{C_{2\varphi}^2\} - E^2\{C_{2\varphi}\} \\ =& \int_{-\infty}^{+\infty} \cos^2(2\varphi) f(\varphi) d\varphi \\ =& 0.5 \end{aligned} \tag{24}$$

V. Performance of Phase Lock Detector

1. Mean and variance analysis on PLD

The theory characteristics have been derived under low/high SNR in above Equations. Fig.1 and Fig.2 show the relationship among C/N_0, mathematical expectation and variance of PLD. The theoretical result is also compared with simulation one from the software receiver[1] and real data. In real data, the PLL will be affected by oscillator noise and user motion when coherent time is long. As seen from figures, Eqs.(16), (17), (23), (24) provide good approximation even under low C/N_0.

2. Lock probability of phase lock detector

In order to calculate the lock probability, the distribution of phase lock detector must be obtained. Fig.3 and Fig.4 shows probability density functions of PLD under low/high SNR according to Eqs.(20) and (22).

In order to archive reliable lock indicator, Neyman-Pearson criteria is introduced. Eqs.(20) and (22) can be used to set the threshold of PLD which mainly depends on the C/N_0 of inputted signal. For a given threshold in a given SNR λ, the false alarm probability P_{FA} and lock probability P_D can be determined in Eqs.(25) and (26).

$$\begin{aligned} P_{FA} =& P\{C_{2\varphi} > Th | \text{noise input only}\} \\ =& \int_{Th}^{1} \frac{1}{\pi\sqrt{(1 + x)(1 - x)}} dx \end{aligned} \tag{25}$$

$$\begin{aligned} P_D =& P\{C_{2\varphi} > Th | S/N = \lambda\} \\ =& P\left\{Y = \frac{1 - C_{2\varphi}}{2\sigma^2} < \frac{1 - Th}{2\sigma^2} | S/N = \lambda\right\} \\ =& P_{r,\chi^2}\left(\frac{1 - Th}{2\sigma^2} | S/N = \lambda\right) \end{aligned} \tag{26}$$

where $P_{c,\chi^2}(y)$ represents the right-tail probability of random variable Y follows $\chi^2(1)$. Fig.5 and Fig.6 depict false alarm probability P_{FA} and lock probability P_D relatively. According to Fig.5, the lock probability will increase when the length of coherent integration becomes longer under same C/N_0. The lock probability can be increased by decreasing threshold, but the false alarm probability will also increase under that situation. As a result, setting threshold must take the SNR and false alarm probability into account at same time.

3 Mean time to lose lock

One factor used to evaluate performances of tracking loop is the Mean time to lose Lock (MTLL). The MTLL should be shorter when the signal does not exist, and be longer when signal exists. Over a total time TT $(N < TT)$, the probability $P_W(N)$ of tracking a non-exist signal during time N is shown in Eq.(27).

$$\begin{aligned} P_W(N) =& P\{c_{2\varphi} > Th | \text{no signal, } N \text{ times}\} \\ =& \left(\Pi_{m=0}^{N-1} P_F\right)(1 - P_F) \\ =& P_F^{N-1}(1 - P_F) \end{aligned} \tag{27}$$

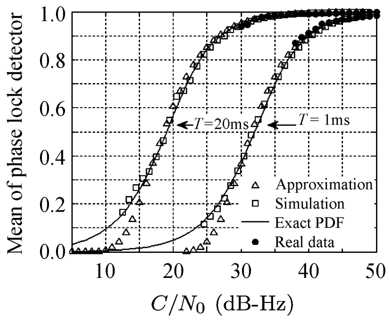

Fig. 1. Mean of PLD under different C/N_0 and integration time

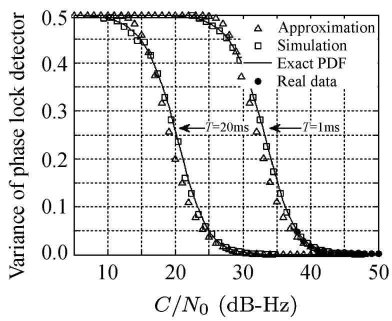

Fig. 2. Variance of PLD under different C/N_0 and integration time

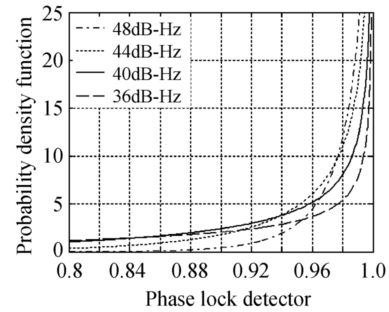

Fig. 3. PDF of PLD under high SNR

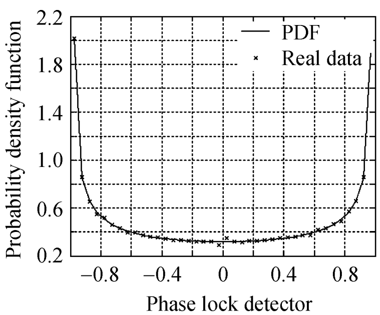

Fig. 4. PDF of PLD under low SNR

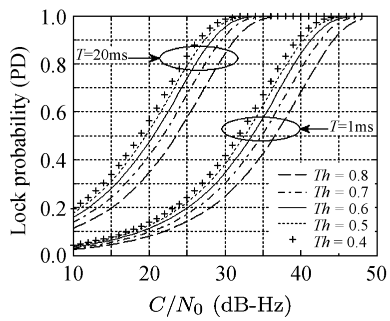

Fig. 5. Lock probability (PD) versus C/N_0 at a given threshold

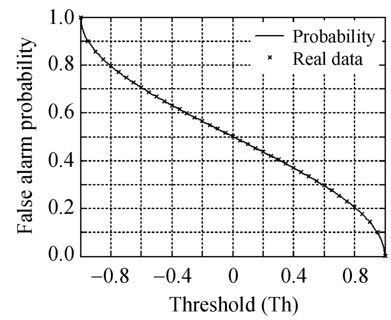

Fig. 6. False alarm probability versus threshold (Th)

where $N = 0, 1, \cdots, \infty$ and the false alarm probability P_F at a given threshold (Th) can be obtained in Eq.(28).

$$P_F = P_{C_{2\varphi}}(c_{2\varphi} > Th) = \int_{Th}^{1} \frac{1}{\pi\sqrt{(1+c_{2\varphi})(1-c_{2\varphi})}} dc_{2\varphi} \tag{28}$$

Based on Eqs.(27) and (28), the MTLL of tracking a non-exist signal can be calculated in Eq.(29).

$$\begin{aligned} E\{T_{MTLL,W}\}|_{TT->\infty} &= \sum_{N=1}^{TT} P_W(N)N|_{TT->\infty} \\ &= \sum_{N=1}^{TT} NP_F^{N-1}(1-P_F)|_{TT->\infty} \\ &= \frac{1}{1-P_F} \end{aligned} \tag{29}$$

Fig.7 shows the relationship between MTLL and threshold setting. When signal does not exist, the MTLL will decline when threshold increasing.

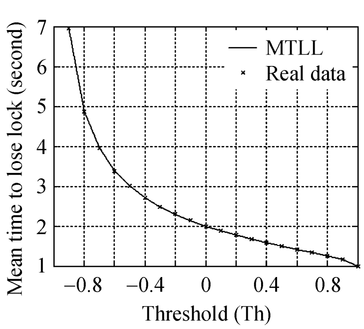

Fig. 7. Mean time of wrong continuous tracking versus threshold

The probability $P_R(N)$ of tracking an existing signal during time N is shown in Eq.(30).

$$\begin{aligned} P_R(N) &= P\{C_{2\varphi} > Th|\text{signal exists, } N \text{ times}\} \\ &= \left(\Pi_{m=1}^{N-1} P_D\right)(1-P_D) \\ &= P_D^{N-1}(1-P_D) \end{aligned} \tag{30}$$

where lock probability P_D at a given threshold can acquired in Eq.(31).

$$\begin{aligned} P_D &= P\{C_{2\varphi} > Th\} \\ &= P\left\{Y = \frac{1-C_{2\varphi}}{2\sigma^2} < \frac{1-Th}{2\sigma^2}\right\} \\ &= P_{c,\chi^2}\left(\frac{1-Th}{2\sigma^2}\right) \end{aligned} \tag{31}$$

The relationship between the mean time to lose lock $T_{MTLL,R}$ and SNR of received signals at a given threshold Th based on Eqs.(30) and (31) infers

$$E\{T_{MTLL,R}\}|_{TT->\infty} = \frac{1}{(1-P_D)} \tag{32}$$

Fig.8 shows relationship between MTLL and C/N_0 under signal tracking period. The MTLL will increase when the SNR or coherent integration time increased.

From results by simulation, the MTLL of non-exist signal is just only 1.5sec at a given threshold $Th = 0.4$. The MTLL is about 80 seconds using 1ms integration under 40dB-Hz signal, and 20ms integration under 27dB-Hz signal. Therefore, an optimal threshold for indicating signal locked or unlocked can be chosen as 0.4 in different situations.

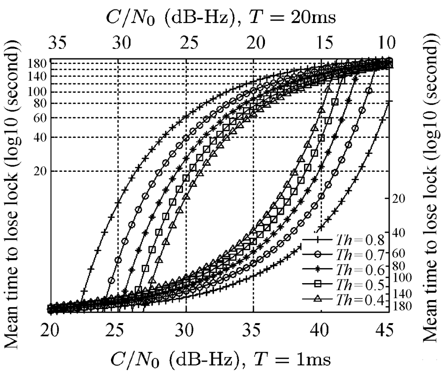

Fig. 8. Mean right time of continuous tracking versus C/N_0

VI. Conclusion and Future Work

In this paper, the theoretical characteristic of the phase lock detector based on narrow-band power has been derived using stochastic analysis. The relationship between probability density function of PLD and C/N_0 has been studied. And locked threshold setting method has been discussed and validated by simulations. Furthermore, the relationship among the mean time to lose lock, threshold setting and C/N_0 has been analyzed. The results show that setting lock threshold to 0.4 maintains 1.5sec average false lock time when signal does not exist, and maintains 80sec average right lock time under 27dB-Hz. The paper proves why that setting satisfied basic need for GPS PLL tracking loop, and provide the theoretical support in lock detector threshold setting.

Recently, using Kalman filter loop instead of traditional DLL/PLL to implement GPS tracking loop has been widely discussed[7]. The new lock mechanism under Kalman filter loop can be further analyzed. The performance of new detector could be compared with the traditional detector based on this research.

References

[1] J.J. Spilker, "GPS signal structure and theoretical performance", *Global Positioning System: Theory and Application*, Vol.1. Ed. Bradford W. Parkinson and James J. Spilker, Jr Washington, DC: American Institute of Aeronautics and Astronautics, pp.390–394, 1996.

[2] A. Mileant and S. Hinedi, "Lock detection in costas loops", *IEEE Trans. Communications*, Vol.40, No.3, pp.480–483, March 1992.

[3] Y. Linn and N. Peleg, "A family of self-normalizing carrier locks detectors and Es/N0 estimators for M-PSK and other phase modulation schemes", *IEEE Trans. Wireless Commun.*, Vol.3, No.5, pp.1659–1668, Sept. 2004.

[4] M.R. Spiegel, *Mathematical Handbook of Formulas and Tables*, Singapore: McGraw-Hill International, 3rd. 2009.

[5] Razavl, Gebre and Akos, "Carrier loop architectures for tracking weak GPS signals", *IEEE Trans. Aerosp. Electron. Syst.*, Vol.44, No.2, pp.697–710, Apr. 2008.

[6] W. Zhuang, "Performance analysis of GPS carrier phase observable", *IEEE Trans. Aerosp. Electron. Syst.*, Vol.30, pp.754–767, Apr. 1996.

[7] J.H. Won, D. Dotterbock and B. Eissfeller, "Performance comparison of different forms of Kalman filter approach for a vector-based GNSS signal tracking loop", *Proceeding of 22nd International Meeting of the Satellite Division of The Institute of Navigation*, Savannah, GA, Sept. 22-25, 2009.

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