Feasibility Conditions for Interference Neutralization in Relay-Aided Interference Channel

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***Abstract*—Interference neutralization (IN) is a new interference management mechanism found from and inherent in interference networks with relays. In this paper, we study the feasibility of IN for relay-aided multi-input-multi-output (MIMO) interference broadcast channel (MIMO-IBC) without symbol extension. Assuming linear transceiver with multiple amplify-and-forward relays, we consider fully connected symmetric systems, where each base station (BS), relay, and user is equipped with multiple antennas, and each user desires multiple data streams. We first present the necessary condition of generalized IN, which is the proper condition to ensure interference-free transmission with linear transceivers for general relay-aided MIMO-IBC. We then find and prove the necessary and sufficient condition of the feasibility of coordinated IN and pure IN for a class of relay-aided MIMO-IBC, where the sufficiency is proved by constructing a full rank coefficient matrix of interference-free transmission equation with sub-matrices of special structures. We show that when each BS and user has the minimal antenna configuration for data transmission, the sufficient and necessary condition for coordinated IN is the same as the necessary condition for generalized IN. When there is an arbitrary number of antennas at the BSs and users, the derived sufficient conditions give rise to the minimum relay configuration required by the coordinated IN and pure IN to support a given number of data streams without interference. Our proof sheds lights on how to use the relay resources to neutralize the interference in an efficient way. The results are applicable for both relay-aided MIMO-IBC and relay-aided interference channel (MIMO-IC).**

***Index Terms*—Interference neutralization, feasibility analysis, DoF, interference channels, relay, MIMO.**

I. Introduction

MULTI-INPUT-MULTI-OUTPUT (MIMO) interference channel (IC) and interference broadcast channel (IBC) refer to setups in cellular networks where multiple multi-antenna base stations (BSs) transmit to multiple multi-antenna users in the same time/frequency resource without data sharing among the BS. In MIMO-IC, each BS communicates with only one user. In MIMO-IBC, each BS communicates with multiple users.

After significant research efforts in the past decades, the capacity region of the interference channel is still unknown. As a first order approximation to the sum capacity in high signal to noise ratio (SNR) regime, the degrees of freedom (DoF) have been extensively studied. Recently, interference alignment (IA) [1], [2] was shown to be able to achieve the information-theoretic maximum DoF of some interference networks. Moreover, it was shown that the overall DoF grows linearly with the cell number B for MIMO-IC and MIMO-IBC with some configurations [2], [3]. Nonetheless, in order to achieve the promised gain, either infinite symbol extensions over time/frequency [2] or a large number of antennas at each user are required [3], which is not realistic. In practice, the achievable DoF does not grow linearly with B in general [4].

Introducing relays to IC provides an effective way of reducing the number of symbol extensions. It was first noted in [5] that a relay-aided three-cell single-input-single-output (SISO)-IC can achieve a DoF of $\frac{3}{2}$ with only two symbol extensions. Similar results were obtained in [6] and [7] for B-cell MIMO-IC with M antennas at each BS and each user, where $\frac{BM}{2}$ DoF can be achieved with a half-duplex relay, or equivalently with two symbol extensions.

When relays are available in IC, on the other hand, it was found in [8] that another interference management technique called interference neutralization (IN) is essential for achieving high DoF. If there are more than one propagation paths from a source to its interfering destination, which is common in relay systems, multiple copies of one interference signal arriving at each user can add up to zero. In other words, the interference can be eliminated in the air, i.e., neutralized. Other techniques with the same idea in essence were independently proposed in the literature, albeit under different names such as distributed orthogonalization in [9] and orthogonalize-and-forward in [10].

The relay-aided IC represents a class of more complicated interference networks. With the possible exception of IA, interference avoiding and cancelation, IN as a new means of interference removal is still not well understood. To find the potential of relay-aided IC with constant coefficients, researchers have investigated the DoF for various settings. Analogous to IC and IBC, the achievable DoF of relay-aided IC with linear transceivers reflects the subspace dimension required to support interference-free transmission. This suggests that the maximum DoF achieved by linear transceivers can be found by analyzing

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the minimum numbers of antennas at the BSs, relays and users that guarantees the linear transceivers to be feasible. Such a feasibility analysis includes finding and proving the necessary and sufficient conditions.

For a relay-aided B-cell SISO-IC where each BS and each user respectively have a single antenna, a maximum of B non-interfering data streams can be transmitted with a single full-duplex B-antenna relay, or with $B(B-1)+1$ half-duplex single-antenna two-hop relays [10]. In [11], the authors considered the SISO-IC with multiple half-duplex relays where the direct links among BSs and users exist. They showed that by using $B(B-2)$ half-duplex relays each with a single antenna, a total of B data streams can be transmitted without interference. Compared with [10], the number of relays is reduced because the direct links are considered. In these scenarios, the interference management scheme for achieving the maximum DoF is pure IN (PIN), where only the relays are employed to eliminate inter-cell interference (ICI). For relay-aided B-cell MIMO-IC where each BS or each user has more antennas than its desired data streams, IA can be employed in conjunction with IN to achieve the maximum DoF. The authors of [12] and [13] introduced aligned interference neutralization for a two-source two-destination relay-aided MIMO-IC, respectively considering one instantaneous relay and two full-duplex relays. The achievable DoF region of the two-source two-destination two-relay aided MIMO-IC was further derived in [14]. For a relay-aided B-cell MIMO-IC where each BS conveys d desired data stream, the authors of [15] obtained a DoF upper bound. The results show that to transmit Bd data streams without interference, the total number of antennas at full-duplex relays needs to exceed Bd and a proper condition originally proposed for MIMO-IC in [16] needs to be satisfied.

While priori results for the achievable DoF in [11]–[14] and the DoF upper bound in [15] provide useful insights to understand the potential of relay-aided IC, for general multi-cell relay-aided MIMO-IC, the maximum achievable DoF remains unknown. The minimum number of antennas at the relays found in [10] only ensures that the interference can be eliminated when applied to the MIMO scenario, but not guarantees that the desired data can be conveyed. The results in [11]–[14] cannot be extended to the setups with more than two cells.

In this paper, we analyze the feasibility of relay-aided MIMO-IBC with constant coefficients, which transmits a target number of non-interfering data streams with linear transceivers. Specifically, we examine the sufficient condition and the necessary condition of using linear transceivers for ensuring interference-free transmission.

We consider fully connected relay-aided MIMO-IBC, where there exist direct links between the BSs and the users. We consider symmetric multi-cell multi-relay networks, where each BS and each user have identical number of antennas and desire for identical number of data streams. For such a general setting, the interference management mechanism may include interference avoidance, interference cancelation, IA and IN, which is referred to as *generalized interference neutralization (GIN)* for simplicity. While analyzing the *necessary and sufficient condition* of GIN feasibility is very challenging, we solve the problem in part by providing the necessary condition for GIN and the sufficient conditions for other two IN strategies, PIN and *coordinated IN (CIN)* defined later.

Our main contributions are as follows. For a large class of relay-aided MIMO-IBC whose system parameters satisfy a mild condition that covers most scenarios in cellular networks, we find and prove the necessary and sufficient conditions for CIN and PIN, from which the achievable DoFs of the networks can be derived. We show that when the networks are with minimal antenna configuration at both BSs and users, the necessary and sufficient condition for CIN coincides with the necessary condition for GIN, from which the maximum achievable DoF can be derived. All previous results in the literature are special cases of ours.

Notations: Transpose and conjugate transpose are represented by $(\cdot)^T$ and $(\cdot)^H$, respectively. $\otimes$ is the Kronecker product operator, $vec(\boldsymbol{A})$ denotes the vectorization of matrix $\boldsymbol{A}$ by concatenating the columns of matrix $\boldsymbol{A}$ into a single column vector. $\boldsymbol{I}_m, \boldsymbol{0}_m$ and $\boldsymbol{0}_{m\times n}$ denote the identity matrix of size $m \times m$, the all zero matrix of size $m \times m$ and the all zero matrix of size $m \times n$, respectively. $\boldsymbol{A}^-$ and $rank(A)$ are the generalized inverse and the rank of matrix $\boldsymbol{A}$. $diag\{\boldsymbol{A}_1, \ldots, \boldsymbol{A}_n\}$ denotes the block diagonal matrix with $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_n$ as its diagonal blocks. $\lceil a \rceil$ and $\lfloor a \rfloor$ are respectively the ceiling and floor functions. $\binom{m}{n}$ is the number of possibilities to choose n numbers out of m.

II. System Model

Consider a downlink network with B coordinating BSs, where each BS intends to transmit d data streams to each of the K users in its own cell. Each BS is equipped with $M \geq Kd$ antennas and each user is with $N \geq d$ antennas. To assist downlink transmission, N_R amplify-and-forward (AF) relays are deployed and each relay has M_R antennas. This system is denoted as the $(B, K, d, M, N, N_R, M_R)$ system for short in the sequel.

We consider fully connected networks, where the link from each BS to each user (called direct link), the link from each BS to each relay (called backhaul link), and the link from each relay to each user (called access link) all exist.

We consider frequency division half-duplex relays, where the relays forward signals in the same frequency band as the direct link but receive in a different frequency band. This is called out-band receiving in-band forwarding relay in literature [17]–[19], which is more desirable than other half-duplex relays in practice [20], [21] because the user only needs to receive in one frequency band. When considering other half-duplex relays, the approach to analyze feasibility condition is similar to the approach in this paper despite that the system model will differ. The system model is illustrated in Fig. 1, where the backhaul link operates in frequency band f_2 and the direct and access links operate in frequency band f_1. The two frequency bands are with identical bandwidth but not overlap.

Denote $\boldsymbol{H}_{b_k,b'} \in \mathbb{C}^{N\times M}$ as the channel matrix from BS b' to the kth user in the bth cell (i.e., user b_k), $\boldsymbol{G}_{b_k,r} \in \mathbb{C}^{N\times M_R}$ as the channel matrix from relay r to user b_k, $\boldsymbol{F}_{r,b} \in \mathbb{C}^{M_R\times M}$ as the channel matrix from BS b to relay r, where $b, b' = 1, \ldots, B, r = 1, \ldots, N_R$ and $k = 1, \ldots, K$.

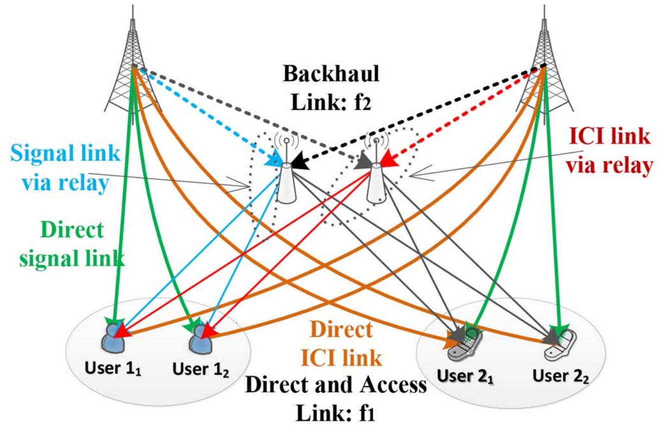


Fig. 1. Example of a two-cell relay-aided MIMO IBC with two users in each cell and two relays in the network, $B=2, K=2, N_R=2$.

All elements in these channel matrices are independent and identically distributed (*i.i.d.*) random variables.

The received signal at user b_k can be expressed as

$$\begin{aligned}\boldsymbol{Y}_{b_k} &= \boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b}\boldsymbol{V}_{b_k}+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_{b_k}\right)\boldsymbol{x}_{b_k}\\ &+\boldsymbol{U}_{b_k}^H\sum_{k'=1,k'\neq k}^{K}\left(\boldsymbol{H}_{b_k,b}\boldsymbol{V}_{b_{k'}}+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_{b_{k'}}\right)\boldsymbol{x}_{b_{k'}}\\ &+\boldsymbol{U}_{b_k}^H\sum_{b'=1,b'\neq b}^{B}\left(\boldsymbol{H}_{b_k,b'}\boldsymbol{V}_{b'}+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}\boldsymbol{F}_{r,b'}\boldsymbol{W}_{b'}\right)\boldsymbol{x}_{b'}\\ &+\boldsymbol{U}_{b_k}^H\left(\boldsymbol{n}_{b_k}+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{n}_r\right), \end{aligned}\quad (1)$$

where $\boldsymbol{x}_{b_k}\in\mathbb{C}^{d\times 1}$ is the symbol vector for user b_k, $\boldsymbol{x}_b=[\boldsymbol{x}_{b_1}^T,\ldots,\boldsymbol{x}_{b_K}^T]^T$ is the symbol vector for the K users in cell b, $\boldsymbol{V}_{b_k}\in\mathbb{C}^{M\times d}$ is the transmit matrix of BS b for user b_k through the direct link, $\boldsymbol{V}_b=[\boldsymbol{V}_{b_1},\ldots,\boldsymbol{V}_{b_K}]\in\mathbb{C}^{M\times Kd}$, $\boldsymbol{W}_{b_k}\in\mathbb{C}^{M\times d}$ is the transmit matrix of BS b for user b_k through the backhaul link, $\boldsymbol{W}_b=[\boldsymbol{W}_{b_1},\ldots,\boldsymbol{W}_{b_K}]\in\mathbb{C}^{M\times Kd}$, $\boldsymbol{U}_{b_k}^H\in\mathbb{C}^{d\times N}$ is the receive matrix at user b_k that is full rank, $\boldsymbol{\Gamma}_r\in\mathbb{C}^{M_R\times M_R}$ is the relay processing matrix at relay r, and $\boldsymbol{n}_r\in\mathbb{C}^{M_R\times 1}$ and $\boldsymbol{n}_{b_k}\in\mathbb{C}^{d\times 1}$ are the noises at relay r and user b_k, respectively.

The first term of (1) is the desired signal of user b_k, which is received from two kinds of propagation links: (1) the *direct signal link* from the BS to the user, whose channel matrix is $\boldsymbol{H}_{b_k,b}$, and (2) the *signal link via relay*, whose effective channel matrix is $\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}$. The second term is the multi-user interference (MUI) at user b_k generated from its master BS when simultaneously transmitting to other users in cell b. Comparing the first and second terms, we can see that the MUI undergoes the same channel as the desired signal. The third term is the ICI at user b_k generated by other BSs when transmitting to their own users. The ICI is also received through two links: (1) the *direct ICI link*, whose channel matrix is $\boldsymbol{H}_{b_k,b'}$, and (2) the *ICI link via relay*, whose effective channel matrix is $\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b'}$ $(b'\neq b)$.

In order to ensure a total number of BKd data streams to be transmitted without interference, the linear transceivers at the BSs, relays and users, $\boldsymbol{W}_{b_k},\boldsymbol{V}_{b_k},\boldsymbol{\Gamma}_r$ and $\boldsymbol{U}_{b_k}^H, b=1,\ldots,B, r=1,\ldots,N_R, k=1,\ldots,K$, should satisfy the following *interference-free transmission constraints*

$$rank\left\{\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b}\boldsymbol{V}_{b_k}+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_{b_k}\right)\right\}=d, \quad (2a)$$

$$\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b}\boldsymbol{V}_{b_{k'}}+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_{b_{k'}}\right)=\boldsymbol{0},\forall b,k'\neq k, \quad (2b)$$

$$\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b'}\boldsymbol{V}_{b'}+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b'}\boldsymbol{W}_{b'}\right)=\boldsymbol{0},\forall k,b'\neq b, \quad (2c)$$

where (2a) is the data transmission constraint to ensure user b_k being able to receive d desired data streams, and (2b) and (2c) are the zero-forcing constraints to remove the MUI and ICI for the user, respectively.

To differentiate the roles of the transceivers in ensuring interference-free transmission, we respectively express $\boldsymbol{V}_b$ and $\boldsymbol{W}_b$ as the concatenation of an inner transmit matrix and an outer transmit matrix as $\boldsymbol{V}_b=\boldsymbol{V}_b^I\boldsymbol{V}_b^O$ and $\boldsymbol{W}_b=\boldsymbol{W}_b^I\boldsymbol{V}_b^O$, where $\boldsymbol{V}_b^O\in\mathbb{C}^{Kd\times Kd}$ is a full rank square matrix, $\boldsymbol{V}_b^I\in\mathbb{C}^{M\times Kd}$ and $\boldsymbol{W}_b^I\in\mathbb{C}^{M\times Kd}$. Similarly, we express $\boldsymbol{U}_{b_k}^H$ as $\boldsymbol{U}_{b_k}^H=(\boldsymbol{U}_{b_k}^O)^H(\boldsymbol{U}_{b_k}^I)^H$, where the outer receive matrix $(\boldsymbol{U}_{b_k}^O)^H\in\mathbb{C}^{d\times d}$ is a full rank square matrix, and the inner receive matrix $(\boldsymbol{U}_{b_k}^I)^H\in\mathbb{C}^{d\times N}$.

Since the desired signal and the MUI received at user b_k undergo the same channel, we can combine (2a) and (2b) into the following constraint to ensure MUI-free data transmission for user b_k,

$$\left(\boldsymbol{U}_{b_k}^O\right)^H\left(\boldsymbol{U}_{b_k}^I\right)^H\left(\boldsymbol{H}_{b_k,b}\boldsymbol{V}_b^I+\sum_{r=1}^{N_R}\boldsymbol{G}_{b_k,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_b^I\right)\boldsymbol{V}_b^O =[\boldsymbol{0},\ldots,\boldsymbol{0},\boldsymbol{\Sigma}_{b_k},\boldsymbol{0},\ldots,\boldsymbol{0}]. \quad (3)$$

Combining the constraints for all users in cell b, we can obtain a compact MUI-free transmission constraints as

$$(\boldsymbol{U}_b^O)^H\left((\boldsymbol{U}_b^I)^H\underbrace{\begin{bmatrix}\boldsymbol{H}_{b_1,b}\\ \vdots\\ \boldsymbol{H}_{b_K,b}\end{bmatrix}}_{\boldsymbol{H}_{b,b}}\boldsymbol{V}_b^I+(\boldsymbol{U}_b^I)^H\sum_{r=1}^{N_R}\underbrace{\begin{bmatrix}\boldsymbol{G}_{b_1,r}\\ \vdots\\ \boldsymbol{G}_{b_K,r}\end{bmatrix}}_{\boldsymbol{G}_{b,r}}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_b^I\right)\boldsymbol{V}_b^O =diag\left\{\boldsymbol{\Sigma}_{b_1},\ldots,\boldsymbol{\Sigma}_{b_K}\right\}, \quad (4)$$

where $(\boldsymbol{U}_b^O)^H\triangleq diag\{(\boldsymbol{U}_{b_1}^O)^H,\ldots,(\boldsymbol{U}_{b_K}^O)^H\}\in\mathbb{C}^{Kd\times Kd}$ is a full rank square matrix, and $(\boldsymbol{U}_b^I)^H\triangleq diag\{(\boldsymbol{U}_{b_1}^I)^H,\ldots,(\boldsymbol{U}_{b_K}^I)^H\}\in\mathbb{C}^{Kd\times KN}$.

As long as the term in the bracket is full rank, we can always design the outer transmit matrix $\boldsymbol{V}_b^O$ and outer receive matrix $(\boldsymbol{U}_b^O)^H$ to ensure (4) satisfied. Therefore, (2a) and (2b) are equivalent to the following constraint,

$$rank\left(\left(\boldsymbol{U}_b^I\right)^H \boldsymbol{H}_{b,b}\boldsymbol{V}_b^I + \left(\boldsymbol{U}_b^I\right)^H \sum_{r=1}^{N_R} \boldsymbol{G}_{b,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_b^I\right) = D, \tag{5}$$

where $D \triangleq Kd$ is the total number of data streams to be transmitted in one cell.

Using the same way, it is not hard to show that the ICI-free constraint (2c) is equivalent to the following constraint,

$$\left(\boldsymbol{U}_{b'}^I\right)^H \boldsymbol{H}_{b',b}\boldsymbol{V}_b^I + \left(\boldsymbol{U}_{b'}^I\right)^H \sum_{r=1}^{N_R} \boldsymbol{G}_{b',r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}\boldsymbol{W}_b^I = \boldsymbol{0}, \tag{6}$$

where the outer transmit matrix $\boldsymbol{V}_b^O$ and outer receive matrix $(\boldsymbol{U}_{b'}^O)^H$ are eliminated because they are full rank square matrices that do not affect the constraint.

From (4), we can see that the role of the outer transmit and receive matrices is to ensure MUI-free. From (5) and (6), we can see that the role of the inner transmit and receive matrices is to ensure data transmission and ICI-free.

To fully employ the resource of the considered system, all the inner transceivers at the BSs, relays and users, $[(\boldsymbol{V}_b^I)^H, (\boldsymbol{W}_b^I)^H], \boldsymbol{\Gamma}_r$ and $\boldsymbol{U}_{b_k}^I, b = 1, \ldots, B, r = 1, \ldots, N_R$ and $k = 1, \ldots, K$, should be involved in removing ICI, where the interference management mechanism may include interference avoidance, interference cancelation, IA and IN. For simplicity, we refer to such a strategy with linear transceiver as *generalized interference neutralization (GIN)*.

From the *necessary and sufficient condition* (i.e., both necessary and sufficient) of GIN feasibility, the maximum achievable DoF of the relay-aided MIMO-IBC with linear transceiver can be derived. To find the feasibility condition of GIN, in the rest of the paper we study the solvability of the interference-free transmission equations (5) and (6).

The major parameters and symbols to be used in this paper are listed in Table I.

TABLE I
MAJOR PARAMETERS AND SYMBOLS

B	Number of cells
K	Number of users in each cell
d	Number of data streams for each user
D	Number of data streams from BS b, $D = Kd$
M	Number of antennas at each BS
N	Number of antennas at each user
N_R	Number of relays
M_R	Number of antennas at each relay
$\boldsymbol{F}_{r,b}$ ($\boldsymbol{F}_{r,b}^w$)	Channel (Effective channel) matrix from BS b to relay r of size $M_R \times M$ ($M_R \times D$)
$\boldsymbol{G}_{b_k,r}$	Channel matrix from relay r to user b_k of size $N \times M_R$
$\boldsymbol{G}_{b,r}^u$	Effective channel matrix from relay r to all the users in cell b of size $D \times M_R$
$\bar{\boldsymbol{G}}_{b,r}^u$	Effective channel matrix from relay r to all users except cell b of size $(B-1)D \times M_R$
$\boldsymbol{H}_{b'_k,b}$	Channel matrix from BS b to user b'_k of size $N \times D$ ($d \times D$)
$\boldsymbol{H}_{b',b}^u$	Effective channel matrix from BS b to all the users in cell b' of size $D \times M$
$\bar{\boldsymbol{H}}_b^u$ ($\bar{\boldsymbol{H}}_b^{uv}$)	Effective channel matrix from BS b to all users in other cells of size $(B-1)D \times M$ ($(B-1)D \times D$)
$\boldsymbol{W}_b$ ($\boldsymbol{W}_b^I$)	Transmit (Inner transmit) matrix of BS b through backhaul link of size $M \times D$
$\boldsymbol{V}_b$ ($\boldsymbol{V}_b^I$)	Transmit (Inner transmit) matrix of BS b through direct link of size $M \times D$
$\boldsymbol{V}_b^O$	Outer transmit matrix of BS b of size $D \times D$
$\boldsymbol{U}_{b_k}$ $(\boldsymbol{U}_{b_k}^I)$	Receive (Inner receive) matrix of user b_k of size $N \times d$
$\boldsymbol{U}_{b_k}^O$	Outer receive matrix of user b_k of size $d \times d$
$\boldsymbol{\Gamma}_r$	Processing matrix of relay r of size $M_R \times M_R$
$\boldsymbol{C}$	Coefficient matrix of *IN equation*
$\hat{\boldsymbol{C}}$	Constructed coefficient matrix of full row rank
$\hat{\boldsymbol{C}}_b$	Sub-matrix of $\hat{\boldsymbol{C}}$ corresponding to cell b
$\hat{\boldsymbol{C}}_b^{m,j}$	Sub-matrix of $\hat{\boldsymbol{C}}_b$ with special structure defined in (22)

III. MAIN RESULTS

In this section, we present the main results on the feasibility analysis for the $(B, K, d, M, N, N_R, M_R)$ relay-aided MIMO-IBC, which includes finding and proving the necessary condition and the sufficient condition.

For MIMO-IC or MIMO-IBC, the feasibility condition of linear IA can be found from analyzing the solvability of a set of multi-variate *quadratic* equations. To find and prove the conditions that are both necessary and sufficient is not easy [4], [22]. For relay-aided MIMO-IBC, the interference-free transmission constraints are multi-variate *cubic* equations, analyzing the *necessary and sufficient condition* of GIN is more challenging.

In fact, the necessary condition (that may not be sufficient) for the relay-aided MIMO-IBC is easy to obtain by extending the result for MIMO-IC, which is the well-known proper condition.

Proper condition was first proposed for MIMO-IC in [16] by relating the linear IA feasibility to the problem of determining the solvability of a system characterized by multivariate polynomial equations, and then was proved to be necessary for IA feasibility of MIMO-IC [4] and MIMO-IBC [22]. A system is proper if the number of independent variables in the interference-free transmission equations is less than that of equations. Because (5) and (6) are multivariate cubic equations, the proper condition is also the necessary condition of GIN feasibility for the relay-aided MIMO-IBC.

For the considered system with frequency division half-duplex relays, the proper condition can be obtained using the same method as in [16]. By counting the number of equations in (5) and (6) and the number of independent variables in $\boldsymbol{U}_b^I, [\boldsymbol{V}_b^I, (\boldsymbol{W}_b^I)^H]$ and $\boldsymbol{\Gamma}_r$, the *proper condition* is obtained as

$$N_R M_R^2 \geq BKd(BKd + d - N - 2M), \tag{7}$$

which is the necessary condition of GIN feasibility.

To ensure that each BS can transmit Kd data streams and each user can receive d data streams, the numbers of antennas at each BS and each user should satisfy

$$M \geq Kd, \quad N \geq d, \tag{8}$$

which is referred to as *minimal antenna configuration*.

Equations (7) and (8) reflect the minimum system resource required for convoying BKD data streams without interference in the network, from which the DoF upper bound of the relay-aided MIMO-IBC can be derived. As discussed in [4], [22], the proper condition may not be the sufficient condition of IA feasibility. This indicates that the DoF upper bound derived from the proper condition may not be achievable.

To analyze the sufficient condition for the relay-aided MIMO-IBC, we linearize the cubic equations by considering two interference coordination strategies. This is one typical methodology to find and prove the sufficient condition [22].

Specifically, when the transmit matrix at each BS for the backhaul link and the receive matrix at each user (i.e., $\boldsymbol{W}_b^I$ and $\boldsymbol{U}_b^I$) do not participate in removing ICI, we call the strategy as the *coordinated interference neutralization (CIN)*.

When the ICI is only eliminated by relays with $\boldsymbol{\Gamma}_r$, we call the strategy as *pure interference neutralization (PIN)*.

In general, the achievable DoF derived from the sufficient condition of PIN feasibility is lower than that from CIN, and both are lower than that from the GIN feasibility. Nonetheless, as shown later, for a special class of systems with minimal antenna configuration, the sufficient condition of CIN feasibility coincides with the necessary condition of GIN feasibility, i.e., it is the necessary and sufficient condition for the GIN.

The following theorems respectively provide the feasibility conditions when considering the two strategies. When each BS has enough antennas to avoid all the ICI, i.e., $M \geq (B-1)D$, it is easy to show that the sufficient and necessary condition of CIN feasibility is

$$N_R M_R \geq BD - M. \tag{9}$$

Therefore, in Theorem 1 we only consider the systems with $M < (B-1)D$.

Theorem 1: For a relay-aided MIMO-IBC $(B, K, d, M, N, N_R, M_R)$ where $M < (B-1)D$, the necessary condition of CIN feasibility is

$$N_R M_R^2 \geq BDJ, \tag{10}$$

and the sufficient condition is also (10) for the systems whose parameters satisfy the following conditions

$$M_R^* R_B \geq DJ, \tag{11a}$$

$$\binom{N_R}{L} \geq \begin{cases} 1, & \frac{D}{n_2} \leq 1, \\ \left\lfloor \frac{D}{D-n_2} \right\rfloor \left\lceil \frac{1}{c} \right\rceil, & \frac{D}{n_2} > 1, \end{cases} \tag{11b}$$

$$\binom{N_R - 1}{L-1} \geq \begin{cases} 1, & \frac{D}{n_2} \leq 1, \\ \left\lfloor \frac{D}{n_2} \right\rfloor \left\lceil \frac{1}{c} \right\rceil + 1, & \frac{D}{n_2} > 1, \end{cases} \tag{11c}$$

where $J \triangleq (B-1)D - M, M_R^* \triangleq \lceil \sqrt{\frac{BDJ}{N_R}} \rceil, L \triangleq \lceil \frac{J}{M_R^*} \rceil, R_B \triangleq \lfloor \frac{N_R M_R^*}{B} \rfloor, n_1 \triangleq J - (L-1)M_R^*, n_2 \triangleq R_B - (L-1)D, c \triangleq \begin{cases} |c_1 - c_2|, & c_1 \neq c_2, \\ 1, & c_1 \triangleq c_2, \end{cases}, c_1 \triangleq max\{\lfloor \frac{D}{n_2} \rfloor, \lceil \frac{D}{D-n_2} \rceil\}$ and $c_2 \triangleq max\{\frac{M_R^*}{n_1}, \frac{M_R^*}{M_R^* - n_1}\}$.

From the definition of M_R^* and L, we can show that $L \sim O(\sqrt{N_R})$. Therefore, with the grow of N_R, both the values of $\binom{N_R}{L}$ and $\binom{N_R-1}{L-1}$ increase. This suggests that when the number of relays increases, the theorem covers more system setups. In fact, it is not hard to show that Theorem 1 applies for most practical cellular networks. For the cases that are not covered by the theorem, simulation results show that M_R^* is still the minimum number of relay antennas required for the $(B, K, d, Kd, d, N_R, M_R^*)$ system with $N_R \geq B$ to be feasible, which unfortunately cannot be proved rigorously.

Corollary 1: When $M = Kd = D$ and $N = d$, both the proper condition in (7) and the necessary condition in (10) reduce to $N_R M_R^2 \geq B(B-2)D^2$.

This indicates that for the systems with minimal antenna configuration whose parameters satisfying (11a), (11b), (11c), the necessary and sufficient condition of CIN coincides with the necessary condition of GIN. In other words, (10) is the necessary and sufficient condition of CIN. For these systems, we can obtain the maximum achievable DoF of the relay-aided MIMO-IBC as follows.

Corollary 2: For a relay-aided MIMO-IBC with $M = D$ and $N = d$, the maximum DoF is

$$DoF \leq \left\lceil \frac{\sqrt{B}}{\sqrt{B-2}} \sqrt{N_R} M_R \right\rceil, \tag{12}$$

which is achievable by GIN for the systems whose parameters satisfy (11a), (11b), (11c).

The corollary implies that in order to increase the DoF, it is more efficient to increase the number of relay antennas rather than increase the number of relays. Specifically, the DoF will grow linearly with B in the following configurations: (1) if N_R does not depend on B but M_R increases with B, i.e., $M_R \sim O(B)$; (2) if $N_R \sim O(B)$ and $M_R \sim O(\sqrt{B-2})$; and (3) if $N_R \sim O(B^2)$ but M_R does not depend on B.

Theorem 2: For the relay-aided MIMO-IBC $(B, K, d, M, N, N_R, M_R)$, the necessary condition of PIN feasibility is

$$N_R M_R^2 \geq BDJ', \tag{13}$$

and the sufficient condition is also (13) for the systems whose parameters satisfy the conditions in the same form as (11a)–(11c) but J and M_R^* being replaced by $J' \triangleq (B-1)D$ and $M_R^{*'} = \lceil \sqrt{\frac{BDJ'}{N_R}} \rceil$.

By comparing the necessary condition for CIN in (10) with that for GIN in (7), we can see how many more antenna resources are required if we do not design the transmit matrix at each BS for the backhaul link and the receive matrix at each user for removing ICI. This is because (7) is for GIN with full coordination among all the transceivers $[(\boldsymbol{V}_b^I)^H, (\boldsymbol{W}_b^I)^H], \boldsymbol{\Gamma}_r$, and $\boldsymbol{U}_b^I, b = 1, \ldots, B, r = 1, \ldots, N_R, k = 1, \ldots, K$.

By comparing the two theorems, it is not hard to show that $J' > J$. From the value of $J' - J$ we can see how many more antenna resources are required if only relays are used for removing ICI.

By comparing Corollary 1 and Theorem 2, we can see that even with the minimal antenna configuration, the CIN still requires fewer antennas than the PIN. This comes from coordinating the transmit matrices at the BSs for direct links and the precessing matrices at the relays in CIN.

For general relay-aided MIMO-IBC with given antenna resources, with the GIN higher DoF might be achieved, but every node in the network needs the channel information of all links, which leads to large training and/or feedback overhead. With the CIN, the achievable DoF is reduced, while the required overhead to obtain channels is also reduced. With the PIN, the achieved DoF is the lowest, yet the required channel information is also minimal. Therefore, the CIN and PIN strategies are of practical interest, despite that they are not optimal in the sense to achieving the maximum DoF of the considered interference network.

IV. Proof of the Main Results

A. Proof of Theorem 1

For the *coordinated interference neutralization*, the transmit matrix at each BS for the backhaul link and the receive matrix at each user are designed for other purpose instead of removing ICI. To analyze the feasibility condition of CIN, we set the inner transmit matrix $\boldsymbol{W}_b^I$ and the inner receive matrix $(\boldsymbol{U}_{b_k}^I)^H$ as arbitrary given matrices. Then, the ICI-free constraints (6) become linear equations of $\boldsymbol{V}_b^I$ and $\boldsymbol{\Gamma}_r$ as

$$\boldsymbol{H}_{b',b}^u \boldsymbol{V}_b^I + \sum_{r=1}^{N_R} \boldsymbol{G}_{b',r}^u \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w = \boldsymbol{0}, \tag{14}$$

where $\boldsymbol{H}_{b',b}^u \triangleq (\boldsymbol{U}_b^I)^H \boldsymbol{H}_{b',b}, \boldsymbol{G}_{b',r}^u \triangleq (\boldsymbol{U}_b^I)^H \boldsymbol{G}_{b',r}$ and $\boldsymbol{F}_{r,b}^w \triangleq \boldsymbol{F}_{r,b} \boldsymbol{W}_b^I$ are the effective channel matrices of the corresponding direct links, access links and backhaul links, respectively.

We further write all the ICI-free constraints related to BS b in (14) in a compact form as follows

$$\bar{\boldsymbol{H}}_b^u \boldsymbol{V}_b^I + \sum_{r=1}^{N_R} \bar{\boldsymbol{G}}_{b,r}^u \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w = \boldsymbol{0}, \tag{15}$$

where

$$\bar{\boldsymbol{H}}_b^u = [(\boldsymbol{H}_{1,b}^u)^T, \ldots, (\boldsymbol{H}_{b-1,b}^u)^T, (\boldsymbol{H}_{b+1,b}^u)^T, \ldots, (\boldsymbol{H}_{B,b}^u)^T]^T \in \mathbb{C}^{(B-1)D \times M}$$

includes all the channel matrices of the direct ICI links from BS b to the users in other cells, $\bar{\boldsymbol{G}}_{b,r}^u = [(\boldsymbol{G}_{1,r}^u)^T, \ldots, (\boldsymbol{G}_{b-1,r}^u)^T, (\boldsymbol{G}_{b+1,r}^u)^T, \ldots, (\boldsymbol{G}_{B,r}^u)^T]^T \in \mathbb{C}^{(B-1)D \times M_R}$ is composed of the channel matrices from relay r to all the users except cell b.

Similarly, the MUI-free transmission constraint (5) becomes

$$rank\left(\boldsymbol{H}_{b,b}^u \boldsymbol{V}_b^I + \sum_{r=1}^{N_R} \boldsymbol{G}_{b,r}^u \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w\right) = D. \tag{16}$$

1) Proof of Necessity: Considering that $M < (B-1)D$, by expressing the matrices $\bar{\boldsymbol{H}}_b^u$ and $\bar{\boldsymbol{G}}_{b,r}^u$ as $\bar{\boldsymbol{H}}_b = [\begin{smallmatrix} \bar{\boldsymbol{H}}_b^{u,1} \\ \bar{\boldsymbol{H}}_b^{u,2} \end{smallmatrix}]$ and $\bar{\boldsymbol{G}}_{b,r}^u = [\begin{smallmatrix} \bar{\boldsymbol{G}}_{b,r}^{u,1} \\ \bar{\boldsymbol{G}}_{b,r}^{u,2} \end{smallmatrix}]$ and considering that $\bar{\boldsymbol{H}}_b^{u,1}$ is a full rank square matrix, we can separate (15) into two equations,

$$\boldsymbol{V}_b^I = -\left(\bar{\boldsymbol{H}}_b^{u,1}\right)^{-1} \sum_{r=1}^{N_R} \bar{\boldsymbol{G}}_{b,r}^{u,1} \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w, \tag{17a}$$

$$\sum_{r=1}^{N_R} \left(\bar{\boldsymbol{G}}_{b,r}^{u,2} - \bar{\boldsymbol{H}}_b^{u,2} \left(\bar{\boldsymbol{H}}_b^{u,1}\right)^{-1} \bar{\boldsymbol{G}}_{b,r}^{u,1}\right) \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w = \boldsymbol{0}, \tag{17b}$$

where $\bar{\boldsymbol{H}}_b^{u,1} \in \mathbb{C}^{M \times M}, \bar{\boldsymbol{H}}_b^{u,2} \in \mathbb{C}^{((B-1)D-M) \times M}, \bar{\boldsymbol{G}}_{b,r}^{u,1} \in \mathbb{C}^{M \times M_R}$ and $\bar{\boldsymbol{G}}_{b,r}^{u,2} \in \mathbb{C}^{((B-1)D-M) \times M_R}$.

These two equations are solvable means that the $(B-1)D$ ICIs generated from BS b can be eliminated, where (17a) and (17b) respectively correspond to M and J ICIs, where $J = (B-1)D-M$. When (17b) is solvable, J ICIs can be eliminated by the relays (i.e., with $\boldsymbol{\Gamma}_r$), and the remaining M ICIs can be jointly eliminated by the relays and BSs (i.e., with $\boldsymbol{\Gamma}_r$ and $\boldsymbol{V}_b^I$).

If the equation in (17b) is solvable, the relay processing matrix $\boldsymbol{\Gamma}_r$ can be obtained, and correspondingly the transmit matrix at the BS for direct link $\boldsymbol{V}_b^I$ can be determined from (17a). Therefore, we only need to analyze the solvability of (17b).

Denote $\bar{\boldsymbol{E}}_{b,r} \triangleq \bar{\boldsymbol{G}}_{b,r}^{u,2} - \bar{\boldsymbol{H}}_b^{u,2}(\bar{\boldsymbol{H}}_b^{u,1})^{-1}\bar{\boldsymbol{G}}_{b,r}^{u,1} \in \mathbb{C}^{((B-1)D-M) \times M_R}$. Using the property of Kronecker product that $vec(\boldsymbol{A}X\boldsymbol{B}) = (\boldsymbol{B}^T \otimes \boldsymbol{A})vec(X)$, the B equations in (17b) can be written as

$$\boldsymbol{C}\boldsymbol{\gamma} = \boldsymbol{0}, \tag{18}$$

where

$$\boldsymbol{C} \triangleq \begin{bmatrix} (\boldsymbol{F}_{1,1}^w)^T \otimes \bar{\boldsymbol{E}}_{1,1} & \cdots & (\boldsymbol{F}_{N_R,1}^w)^T \otimes \bar{\boldsymbol{E}}_{N_R,1} \\ \vdots & & \vdots \\ (\boldsymbol{F}_{1,B}^w)^T \otimes \bar{\boldsymbol{E}}_{B,1} & \cdots & (\boldsymbol{F}_{N_R,B}^w)^T \otimes \bar{\boldsymbol{E}}_{B,N_R} \end{bmatrix} \in \mathbb{C}^{BD((B-1)D-M) \times N_R M_R^2},$$

and $\boldsymbol{\gamma} \triangleq [\,vec(\boldsymbol{\Gamma}_1)^T \;\; \cdots \;\; vec(\boldsymbol{\Gamma}_{N_R})^T\,]^T \in \mathbb{C}^{N_R M_R^2 \times 1}$ is composed of the unknown variables in the relay processing matrices.

From the result in [23] we know that (17b) has full rank solutions (i.e., the solution of $\boldsymbol{\Gamma}_r$ will be full rank) *iff* $\boldsymbol{C}$ is full row rank. When $\boldsymbol{\Gamma}_r$ is full row rank, $\boldsymbol{V}_b^I$ obtained from (17a) is full rank with probability one. Then, the rank constraint (16) is satisfied because $\boldsymbol{H}_{b,b}^u$ in the first term is independent of the other terms.

As a result, in order to ensure the constraints in (15) and (16) satisfied, $\boldsymbol{C}$ must be a full row rank fat matrix, i.e.,

$$rank(\boldsymbol{C}) = BD((B-1)D-M), \tag{19}$$

which proves that the necessary condition is (10).

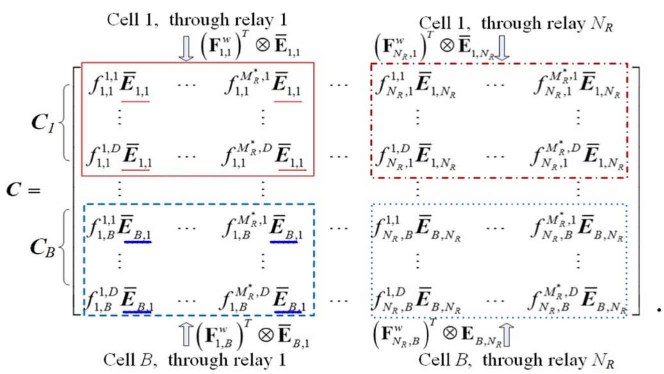
2) Proof of Sufficiency: To prove the sufficient condition, we only need to prove that for the $(B, K, d, M, N, N_R, M_R^*)$ system with $M_R^* = \lceil\sqrt{\frac{BDJ}{N_R}}\rceil$ (that is the minimum number of relay antennas satisfying the necessary condition (10)), if the conditions (11a)–(11c) are satisfied, the matrix $\boldsymbol{C}$ will be full row rank with probability one.

From the definition we know that $\boldsymbol{C}$ is composed of multiple Kronecker products of channel matrices, whose rank is hard to find. We use an alternative approach to prove by constructing a full row rank coefficient matrix.

From the results of Theorem 2 in [24] we know that if $\boldsymbol{C}$ is full row rank for any given values of $\boldsymbol{F}_{r,b}^w$ and $\bar{\boldsymbol{E}}_{b,r}$, $\boldsymbol{C}$ will be full rank for any matrices $\boldsymbol{F}_{r,b}^w$ and $\bar{\boldsymbol{E}}_{b,r}$ with *i.i.d* entries. The dimension of the matrices $\boldsymbol{F}_{r,b}^w$ and $\bar{\boldsymbol{E}}_{b,r}$ in $\boldsymbol{C}$ are respectively $D \times M_R^*$ and $J \times M_R^*$.

In the sequel, we first construct the coefficient matrix as a block diagonal matrix by setting "0"s and "1"s in $\boldsymbol{F}_{r,b}^{w}$ and setting the elements in $\bar{\boldsymbol{E}}_{b,r}$ as arbitrary *i.i.d.* variables, and then construct each block diagonal matrix. Finally, we prove that the constructed coefficient matrix are full row rank with probability one.

We start by observing the structure of the coefficient matrix. We rewrite it in a more detailed form with $f_{r,b}^{i,j}$ denoting the effective channel coefficient from the jth antenna of BS b to the ith antenna of relay r, which includes B row blocks as follows



Unlike MIMO-IC, the coefficient matrix of the relay-aided MIMO-IBC is not a sparse matrix, and hence the method of finding the non-singular Jacobin matrix in [4] is not applicable.

The coefficient matrix $\boldsymbol{C}$ is composed of B blocks $\boldsymbol{C}_b = [(\boldsymbol{F}_{1,b}^{w})^T \otimes \bar{\boldsymbol{E}}_{b,1} \ \cdots \ (\boldsymbol{F}_{N_R,b}^{w})^T \otimes \bar{\boldsymbol{E}}_{b,N_R}], b = 1,\ldots,B$. The structure of matrix $\boldsymbol{C}$ suggests that if the non-zero columns in $(\boldsymbol{F}_b^w)^T = [(\boldsymbol{F}_{1,b}^{w})^T \ \cdots \ (\boldsymbol{F}_{N_R,b}^{w})^T]$ and $(\boldsymbol{F}_{b'}^w)^T, b \neq b'$ do not overlap, the non-zero columns of $\boldsymbol{C}_b$ and $\boldsymbol{C}_{b'}$ will not overlap. For simplicity and easy understanding, we set these non-zero columns as uniform as possible in the N_R submatrices of $(\boldsymbol{F}_b^w)^T$. Considering that $(\boldsymbol{F}_b^w)^T$ has $N_R M_R^*$ columns, each $(\boldsymbol{F}_b^w)^T$ can have at most R_B non-zero columns not overlapping with $(\boldsymbol{F}_{b'}^w)^T$. Specifically, denote $p \triangleq \lfloor \frac{R_B}{N_R} \rfloor$ and $q \triangleq R_B - pN_R$. For the sub-matrices $(\boldsymbol{F}_{r,b}^{w})^T, r = 1,\ldots,N_R$, let q of them have $p+1$ non-zero columns and the rest $N_R - q$ matrices have p non-zero columns. In this way, we can construct a matrix of $(\boldsymbol{F}_b^w)^T$ with the following structure

$$\left(\hat{\boldsymbol{F}}_b^w\right)^T = \Big[\underbrace{\left(\hat{\boldsymbol{F}}_{1,b}^{w}\right)^T}_{(p+1)} \cdots \underbrace{\left(\hat{\boldsymbol{F}}_{q,b}^{w}\right)^T}_{(p+1)} \ \underbrace{\left(\hat{\boldsymbol{F}}_{q+1,b}^{w}\right)^T}_{p} \cdots \underbrace{(\hat{\boldsymbol{F}}_{N_R,b}^{w})^T}_{p}\Big],$$

where the first q matrices have $p+1$ non-zero columns and the rest have p non-zero columns, therefore the total number of non-zero columns is $q(p+1) + (N_R - q)p = R_B$. Actually, it does not matter which matrices have $p+1$ and which have p non-zero columns.

Then, by reorganizing the columns, we can rewrite the non-zero blocks in each row block of $\boldsymbol{C}$ as shown in (20) at the bottom of the page, which includes D row sub-blocks and R_B column sub-blocks of $\bar{\boldsymbol{E}}_{b,r}$. Since the size of $\bar{\boldsymbol{E}}_{b,r}$ is $J \times M_R^*$, condition (11a) ensures that $\hat{\boldsymbol{C}}_b$ is a fat matrix.

In this way, the coefficient matrix is decoupled into a block diagonal matrix

$$\hat{\boldsymbol{C}} = diag\left\{\hat{\boldsymbol{C}}_1,\ldots,\hat{\boldsymbol{C}}_B\right\}, \tag{21}$$

whose rank is determined by the sum of the rank of each $\hat{\boldsymbol{C}}_b$.

In the sequel, we construct a full rank matrix $\hat{\boldsymbol{C}}_b$. We begin with introducing a sub-matrix of $\hat{\boldsymbol{C}}_b$ with special structure. We proceed to show that if such a sub-matrix is constructed following three rules the sub-matrix will be full row rank. Finally, we show that when conditions (11a)–(11c) are satisfied, $\hat{\boldsymbol{C}}_b$ can be constructed as a block diagonal matrix composed of several full rank sub-matrices.

Define a sub-matrix of $\hat{\boldsymbol{C}}_b$ with the structure shown in (22) at the bottom of the page, where $\boldsymbol{P}_{i,b} = [\bar{\boldsymbol{E}}_{b,r_1^i} \ \cdots \ \bar{\boldsymbol{E}}_{b,r_{L-1}^i}], i = 1,\ldots,m$, are

$$\hat{\boldsymbol{C}}_b = \begin{bmatrix} f_{1,b}^{1,1}\bar{\boldsymbol{E}}_{b,1}\cdots f_{1,b}^{p+1,1}\bar{\boldsymbol{E}}_{b,1} & \cdots & f_{N_R,b}^{1,1}\bar{\boldsymbol{E}}_{b,N_R}\cdots f_{N_R,b}^{p,1}\bar{\boldsymbol{E}}_{b,N_R} \\ \vdots & & \vdots \\ \underbrace{f_{1,b}^{1,D}\bar{\boldsymbol{E}}_{b,1}\cdots f_{1,b}^{p+1,D}\bar{\boldsymbol{E}}_{b,1}}_{\left(\hat{\boldsymbol{F}}_{1,b}^{w}\right)^T\otimes\bar{\boldsymbol{E}}_{b,1}} & \cdots & \underbrace{f_{N_R,b}^{1,D}\bar{\boldsymbol{E}}_{b,N_R}\cdots f_{N_R,b}^{p,D}\bar{\boldsymbol{E}}_{b,N_R}}_{(\hat{\boldsymbol{F}}_{N_R,b}^{w})^T\otimes\bar{\boldsymbol{E}}_{b,N_R}} \end{bmatrix}, \tag{20}$$

$$\hat{\boldsymbol{C}}_b^{m,j} = \left[\begin{array}{ccc|ccc} \boldsymbol{P}_{1,b} & & & \bar{\boldsymbol{E}}_{b,r_L^1} & & \\ & \ddots & & & \ddots & \\ & \boldsymbol{P}_{j,b} & & & & \bar{\boldsymbol{E}}_{b,r_L^j} \\ \hline & \boldsymbol{P}_{j+1,b} & & \bar{\boldsymbol{E}}_{b,r_L^1} & \cdots & \bar{\boldsymbol{E}}_{b,r_L^j} \\ & & \ddots & \vdots & & \vdots \\ & & \boldsymbol{P}_{m,b} & \bar{\boldsymbol{E}}_{b,r_L^1} & \cdots & \bar{\boldsymbol{E}}_{b,r_L^j} \end{array}\right], \tag{22}$$

composed of $L-1$ block matrices each with size $J \times M_R^*$, and $r_1^i, \ldots, r_{L-1}^i$ denote the relay indexes. We refer to $\{r_1^i, \ldots, r_{L-1}^i\}$ as a relay index set of $\boldsymbol{P}_{i,b}$.

Such a sub-matrix can be obtained from $\hat{\boldsymbol{C}}_b$ by setting some of the scalars "f" in (20) as "0" or "1". The number of row blocks in $\hat{\boldsymbol{C}}_b^{m,j}$ is m and $m \leq D$, and the number of column blocks is $m(L-1)+j$ and $m(L-1)+j \leq R_B$.

In order to be full row rank, $\hat{\boldsymbol{C}}_b^{m,j}$ should be a fat matrix. Then, its numbers of rows and columns should satisfy $mJ \leq [m(L-1)+j]M_R^*$, which amounts to the following relationship

$$\frac{m}{j} \leq \frac{M_R^*}{J-(L-1)M_R^*} \triangleq \frac{M_R^*}{n_1}, \tag{23}$$

where $n_1 = J-(L-1)M_R^*$, which was defined in Theorem 1 and is rewritten here for convenience.

The following Lemma provides the construction rules to ensure sub-matrix $\hat{\boldsymbol{C}}_b^{m,j}$ to be full row rank with probability one. Since the way to construct the sub-matrix for each cell b is the same, we therefore neglect the cell index b in the block matrices inside $\hat{\boldsymbol{C}}_b^{m,j}$ for conciseness.

Lemma 1: The sub-matrix $\hat{\boldsymbol{C}}_b^{m,j}$ will be full row rank if it is constructed with the following three rules:

Rule 1: In each of the first j row blocks of $\hat{\boldsymbol{C}}_b^{m,j}$, the relay indexes of the L non-zero blocks of "$\bar{\boldsymbol{E}}$" differ and the L indexes in different row blocks are not all the same.

Rule 2: In each of the last $m-j$ row blocks of $\hat{\boldsymbol{C}}_b^{m,j}$, the first $L-1$ relay indexes are different, and these $L-1$ indexes in different row blocks are not all the same.

Rule 3: For $1 \leq i \leq j$ and $j+1 \leq i' \leq m$, if the relay index r_L^i in $\bar{\boldsymbol{E}}_{b,r_L^i}$ in the ith row block is the same as one of the relay indexes in $\boldsymbol{P}_{i'}$, or if the $L-1$ relay indexes in $\boldsymbol{P}_i$ are all the same as those in $\boldsymbol{P}_{i'}$, the block $\bar{\boldsymbol{E}}_{b,r_L^i}$ in the i'th row block can be transformed to a block of zero by elementary transformation. We refer to this kind of block as "*erasable*" block. Otherwise, the block is "*unerasable*". In the lower right corner of $\hat{\boldsymbol{C}}_b^{m,j}$, there should be at least one "*unerasable*" block in each row block and each column block, and for each "*unerasable*" block, there is at least another one "*unerasable*" block in either the same row or the same column block.

Proof: See Appendix A. ■

According to *Rule 1*, all the L relay indexes in the first j row blocks of $\hat{\boldsymbol{C}}_b^{m,j}$ should be different and they are not all the same to each other. This requires that $L \leq N_R$ and $\binom{N_R}{L} \geq j$.

According to *Rule 2*, the $L-1$ relay indexes from the $(j+1)$th to the mth row blocks are not all the same. This requires that $\binom{N_R}{L-1} \geq m-j$.

According to *Rule 3*, there is at least one relay index in $r_L^1, \ldots, r_L^j$ that is different from all the $L-1$ relay indexes in $\boldsymbol{P}_{j+1}, \ldots, \boldsymbol{P}_m$. This requires that $\binom{N_R-1}{L-1} \geq m-j+1$.

Consequently, if we can construct a sub-matrix that satisfies these requirements, the sub-matrix will be full row rank.

From the definition $L = \lceil \frac{J}{M_R^*} \rceil$, we know that

$$L < \frac{J}{M_R^*} + 1. \tag{24}$$

From the definition $M_R^* = \lceil \sqrt{\frac{BDJ}{N_R}} \rceil$, we know that $M_R^* \geq \sqrt{\frac{BDJ}{N_R}}$. Upon substituting into (24), we have

$$L < \frac{J}{\sqrt{\frac{BDJ}{N_R}}} + 1 = \sqrt{N_R \frac{J}{BD}} + 1 < \sqrt{N_R} + 1.$$

Since $N_R \geq 1$, it follows that $L < N_R$ is always satisfied.

Considering that $\binom{N_R}{L} \geq \binom{N_R}{L-1}$, as long as $\binom{N_R-1}{L-1} \geq m-j+1$ is satisfied, $\binom{N_R}{L-1} \geq m-j$ will always be satisfied.

This suggests that if we can construct a sub-matrix $\hat{\boldsymbol{C}}_b^{m,j}$ satisfying the following conditions, $\hat{\boldsymbol{C}}_b^{m,j}$ will be full row rank,

$$\binom{N_R}{L} \geq j, \tag{24a}$$

$$\binom{N_R-1}{L-1} \geq m-j+1. \tag{24b}$$

As shown from (24b), when the number of row blocks in $\hat{\boldsymbol{C}}_b^{m,j}$, m, is small, the condition is more easily to be satisfied. Based on this observation, we divide $\hat{\boldsymbol{C}}_b$ into multiple smaller sub-matrices with fewer row blocks.

The expressions of the left-hand side of (24a) and (24b) are the same with those in (11b) and (11c) in Theorem 1. Therefore, if we can find the values of m and j that satisfy (24a) and (24b), and show that if (11b) and (11c) satisfy then conditions (24a) and (24b) will satisfy, then we can construct a sub-matrix $\hat{\boldsymbol{C}}_b^{m,j}$ of full row rank. This is exactly what we will do in the proof of the next lemma.

Lemma 2: For any system whose parameters satisfy conditions (11b) and (11c), we can construct $\hat{\boldsymbol{C}}_b$ as a block diagonal matrix as follows

$$\hat{\boldsymbol{C}}_b = diag\left\{\hat{\boldsymbol{C}}_b^{m,j}, \ldots, \hat{\boldsymbol{C}}_b^{m',j'}\right\}, \tag{25}$$

where all the diagonal blocks are full row rank.

Proof: See Appendix B. ■

Since each diagonal block of the constructed coefficient matrix $\hat{\boldsymbol{C}}$ is full row rank, $\hat{\boldsymbol{C}}$ is full row rank.

This completes the proof of Theorem 1. ■

B. Proof of Theorem 2

For the *pure interference neutralization*, only relays are employed for removing ICI. To analyze the feasibility of the PIN, we set all the inner transmit or receive matrices $\boldsymbol{W}_b^I, (\boldsymbol{U}_{b_k}^I)^H$ and $\boldsymbol{V}_b^I$ as arbitrary. Then, the interference-free transmission constraints (5) and (6) become linear functions of $\boldsymbol{\Gamma}_r$, which are

$$rank\left(\boldsymbol{H}_{b,b}^{uv} + \sum_{r=1}^{N_R} \boldsymbol{G}_{b,r}^u \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w\right) = D, \tag{25a}$$

$$\boldsymbol{H}_{b',b}^{uv} + \sum_{r=1}^{N_R} \boldsymbol{G}_{b',r}^u \boldsymbol{\Gamma}_r \boldsymbol{F}_{r,b}^w = \boldsymbol{0}, \tag{25b}$$

where $\boldsymbol{H}_{b',b}^{uv} \triangleq \boldsymbol{H}_{b',b}^u \boldsymbol{V}_b^I \in \mathbb{C}^{D \times D}$.

The compact form of ICI-free constraints (25b) related to BS b is therefore

$$\bar{\boldsymbol{H}}_b^{uv} + \sum_{r=1}^{N_R} \bar{\boldsymbol{G}}_{b,r}\boldsymbol{\Gamma}_r\boldsymbol{F}_{r,b}^{w} = \boldsymbol{0}, \tag{26}$$

where $\bar{\boldsymbol{H}}_b^{uv} = [(\boldsymbol{H}_{1,b}^{uv})^T, \ldots, (\boldsymbol{H}_{b-1,b}^{uv})^T, (\boldsymbol{H}_{b+1,b}^{uv})^T, \ldots, (\boldsymbol{H}_{B,b}^{uv})^T]^T \in \mathbb{C}^{(B-1)D\times D}$.

Again, by using the property of Kronecker product, the ICI-free equation can be expressed as

$$\boldsymbol{C}\boldsymbol{\gamma} = -\boldsymbol{h}, \tag{27}$$

where

$$\boldsymbol{C} \triangleq \begin{bmatrix} (\boldsymbol{F}_{1,1}^{w})^T \otimes \bar{\boldsymbol{G}}_{1,1} & \cdots & (\boldsymbol{F}_{N_R,1}^{w})^T \otimes \bar{\boldsymbol{G}}_{1,N_R} \\ \vdots & & \vdots \\ (\boldsymbol{F}_{1,B}^{w})^T \otimes \bar{\boldsymbol{G}}_{B,1} & \cdots & (\boldsymbol{F}_{N_R,B}^{w})^T \otimes \bar{\boldsymbol{G}}_{B,N_R} \end{bmatrix} \in \mathbb{C}^{B(B-1)D^2\times N_R M_R^2}, \boldsymbol{\gamma}$$

is defined as in (18) and $\boldsymbol{h} = [\,vec(\bar{\boldsymbol{H}}_1^{uv})^T, \ldots, vec(\bar{\boldsymbol{H}}_B^{uv})^T\,]^T$.

The elements in the first term $\boldsymbol{H}_{b,b}^{uv}$ in (25a) are *i.i.d*, it is full rank almost surely. Since it is independent of other terms in the equation, the MUI-free transmission constraint is automatically satisfied. As a result, the feasibility only relies on the solvability of (27).

According to linear algebra, (27) is solvable *iff* $rank(C) = rank([\boldsymbol{C}, -\boldsymbol{h}])$. Because all the elements in $\boldsymbol{h}$ are independent from those in $\boldsymbol{C}$, this is equivalent to requiring that $\boldsymbol{C}$ is full row rank. Therefore, the necessary and sufficient condition for the solvability of (27) is

$$rank(\boldsymbol{C}) = B(B-1)D^2. \tag{28}$$

The necessary condition in (13) is now proved.

The proof of sufficiency is the same as in Theorem 1 by setting $J' = (B-1)D$ and $\bar{\boldsymbol{E}}_{b,r} = \bar{\boldsymbol{G}}_{b,r}$. ■

V. DISCUSSIONS

In this section, we strive to explain the intuitive meaning implied by the proof of the main results as well as the parameters appeared in the theorems, and show the connection of our results with existing results in the literature.

A. Understanding the Main Results

1) Understanding the Two Strategies: For the *PIN*, we have shown that the ICI-free constraint is (27), from which we can explain the name for this strategy. Each element of vector $\boldsymbol{h}$ is the channel coefficient from one transmit antenna at a BS to one receive antenna at a user in other cells. Each element of $\boldsymbol{C}\boldsymbol{\gamma}$ is the effective channel coefficient of the *ICI link via relay* between the same pair of transmit and receive antennas. (27) indicates that the ICIs received at the user through the direct ICI link and through the relays have the same amplitude but opposite signs, such that these two copies are removed "in the air", i.e., are neutralized. Because all the ICIs are neutralized by the relays, where a B-cell network degrades to B isolated multi-user MIMO systems, the strategy is pure IN.

For the *CIN*, however, although (18) looks similar to (27), from which we cannot see how the ICI is removed because $\boldsymbol{C}\boldsymbol{\gamma}$ is no longer the effective channel coefficient of the *ICI link via relay*. Nonetheless, comparing (15) with (26), we can see that by coordinating the transmit matrices at the BSs for direct links and the processing matrices at the relays, the ICIs received at the user through the direct ICI link and through the relays are neutralized.

2) Understanding How the Relay Antennas are Used: From the way to construct $\hat{\boldsymbol{C}}_b^{m,j}$, we can reveal how the relay resources are used to neutralize the ICI.

Since each column block of $\hat{\boldsymbol{C}}_b$ corresponds to a relay antenna, and each row block corresponds to the ICI generated to one user, the constructed $\hat{\boldsymbol{C}}_b$ denotes that if $\bar{\boldsymbol{E}}_{b,r}$ appears in one row block of $\hat{\boldsymbol{C}}_b$ (i.e., the corresponding $f_{r,b}^{i,j} = 1$), one antenna of relay r will be involved in eliminating the ICI generated to one user in cell b.

As shown in the proof of Lemma 1, the sub-matrices can be divided into four types, which reflect different ways that the relay antennas are used to neutralize the ICI.

- When $m = j = 1$, the sub-matrix reduces to ***Type I*** matrix shown in (A.1). When the coefficient matrix only contains the type I matrices each $\bar{\boldsymbol{E}}_{b,r}$ only appears once in the same column block of $\hat{\boldsymbol{C}}$. It means that if one antenna of a relay is used to help remove the ICI generated to a specific user, the antenna will not be used to eliminate the ICI generated to other users. Such kind of relay antenna is called *"private antenna"*.
- When $m > 1, j = 1$, the sub-matrix reduces to ***Type II*** matrix shown in (A.2). The matrix $\bar{\boldsymbol{E}}_{r_L^1}$ appears in all row blocks, which means that the corresponding relay antenna is involved in eliminating the ICI generated to all the m users. Such kind of relay antenna is called *"shared antenna"*.
- When $j = m-1$, the sub-matrix reduces to ***Type III*** matrix shown in (A.13). In this scenario, one relay antenna is shared by two users and the last user shares one antenna with each of the previous $m-1$ users.
- For the case of $1 < j < m-1$, the sub-matrix has the general form shown in (22) that is defined as ***Type IV*** matrix. In this scenario, the relay antennas are used in a hybrid way for those of type I, type II and type III matrices.

From the procedure of proof, we know that J is the number of ICIs generated to one data stream that needs to be eliminated by relays. M_R^* is the minimum number of antennas at each relay that satisfies the necessary condition $N_R M_R^2 \geq BDJ$. Since there are BD data streams in the system and each data stream experiences J ICIs, the term on the right-hand side of the inequality denotes the total number of ICIs, and the term on the left-hand side is the number of variables provided by the N_R relays. Therefore M_R^* can also be interpreted as the number of variables provided by each relay antenna. Then, L can be viewed as the minimum number of relay antennas required by each data stream to eliminate the J ICIs. Since there are $N_R M_R^*$ relay antennas in the system, R_B can be viewed as the number of relay antennas uniformly allocated for each cell. The condition (11a), $M_R^* R_B \geq DJ$, indicates that the number of variables in each cell should exceed the number of ICIs in the same cell.

From the proof, the parameter n_1 can be viewed as the number of variables required from one relay antenna besides the $(L-1)M_R^*$ variables provided by the $L-1$ "private" antennas to eliminate the J ICIs at one data stream, and n_2 can be regarded as the total number of "shared" relay antennas in each cell.

When $\frac{M_R^*}{n_1} > \frac{M_R^*}{M_R^*-n_1}$ and $\frac{M_R^*}{n_1} > c_1$ (corresponding to Case 1.2 in the proof of Lemma 2), only a small fraction of the variables provided by a "shared" relay antenna are required to eliminate the ICI to one data stream. In this case, the antenna can be shared with many data streams, and $\hat{\boldsymbol{C}}$ is constructed only with type II matrix. This represents one extreme case with minimum number of *"shared antennas"*: multiple users in each cell "share" a single relay antenna.

When $\frac{M_R^*}{n_1} < \frac{M_R^*}{M_R^*-n_1}$ and $\frac{M_R^*}{n_1} < c_1$ (corresponding to Case 2.2 in the proof of Lemma 2), on the other hand, a large fraction of the variables for eliminating the ICI to one data stream needs to be provided by a "shared" relay antenna. In this case, although the antenna is shared by two data streams, the variables it provided is only sufficient enough for eliminating the ICI of one data stream, for another data stream sharing the same antenna, more variables from other "shared" antennas are necessary. The coefficient matrix $\hat{\boldsymbol{C}}$ of such a system can be constructed only with type III matrices. This represents another extreme case with maximum number of *"shared antennas"*: each of the first $K-1$ users "shares" one antenna with the Kth user.

When $\frac{M_R^*}{n_1}$ is with other values (corresponding to Cases 1.3 and 2.3 in the proof of Lemma 2), the coefficient matrix of the system is constructed with both types II and IV matrices or both types III and IV matrices. This is a scenario in between the two extreme cases.

When $n_2 \geq D$ (corresponding to Cases 1.1 and 2.1 in the proof of Lemma 2), $\hat{\boldsymbol{C}}$ is constructed only with type I matrices $\hat{\boldsymbol{C}}_b^{1,1}$. In this case, the overall number of relay antennas is large enough such that all the data streams only need the *"private antennas"* to remove their ICIs. In other words, the relays can provide different antennas to neutralize each ICI. For this kind of systems, the sufficiency proof is significantly simplified as shown in the proof of Lemma 1.

B. Relation With Existing Results

In the sequel, we list the class of systems satisfying the conditions in Theorem 1 and Theorem 2, and show that existing results in literature are all special cases of ours. In fact, for all the systems considered in existing works, proving the sufficient condition is much simpler, because the coefficient matrices can be constructed only with the type I matrices.

For the class of systems with $M = nD, N = d, N_R = B(B-n-1)$, and $M_R^* = D$, it is not hard to show that the conditions (11a)–(11c) in Theorem 1 automatically satisfy. This indicates that these systems can transmit without interference by using CIN *iff* (10) is true. When $n = 1$ and $D = 1$, our result is the same as that presented in [11] for SISO-IC, which is a special case of Corollary 1.

For the class of systems with the following configurations, it is not hard to show that the conditions (11a)–(11c) in Theorem 2 automatically satisfy (recall that M_R^* of the two theorems are different and thus the conditions differ). This indicates that these systems can transmit without interference by using PIN *iff* (13) is true:

- $N_R = nB, B = 1 + nl^2, \frac{D}{l} = z$ and $M_R^* = lD$, or $N_R = n(B-1), B = nl^2, \frac{D}{l} = z$, and $M_R^* = lD$, where n, l and z are arbitrary integers.
- $N_R = B(B-1)D^2, M_R^* = 1$. When $D = 1$, our result reduces to $N_R = B(B-1)$, which is higher than that obtained for a two-hop relay-aided SISO-IC in [10], whose required minimum number of relays is $B(B-1)+1$. This is because we consider the direct links among the BSs and users.
- $B = 2, N_R = 2, M_R^* = D$. For such a two-cell two-relay setting, overall $2D$ data streams can be transmitted without interference according to our results. Under the two-cell two-hop two-relay setting, $2D-1$ data streams can be transmitted as reported in [13] and [14]. Again, the difference comes from the direct links we considered.

In summary, we have obtained the sufficient condition of feasibility for a much wider class of systems than priori works, which are reflected in requiring less minimum system resources or supporting more interference-free data streams.

VI. CONCLUSION

In this paper, we analyzed the feasibility of interference neutralization for fully connected relay-aided MIMO-IBC without symbol extension. We derived the necessary and sufficient condition for a class of relay-aided MIMO-IBC with coordinated and pure interference neutralization, where the sufficiency was proved by constructing a full row rank coefficient matrix of interference-free transmission equation with special structure. We further showed that for the system with minimal antenna configuration at each BS and user, the sufficient condition for coordinated interference neutralization coincides with the necessary condition of generalized interference neutralization, from which the maximum achievable DoF can be derived. For the systems with more antennas at each BS or each user, from the provided sufficient condition we can derive the achievable DoF. All system settings considered in the literature are special cases of ours. Our conclusions are applicable to both relay-aided MIMO-IBC and MIMO-IC.

APPENDIX A
PROOF OF LEMMA 1

Proof: The sub-matrix $\hat{\boldsymbol{C}}_b^{m,j}$ reduces to several special types depending on the relationship of m and j. In the sequel, we prove that when constructed following the three rules, each type of the sub-matrix will be full row rank.

Type I: When $m = j = 1$, the sub-matrix reduces to

$$\hat{\boldsymbol{C}}_b^{1,1} = \left[\boldsymbol{P}_1, \bar{\boldsymbol{E}}_{r_L^1} \right], \tag{A.1}$$

whose size is $J \times LM_R^*$.

Since $L = \lceil \frac{J}{M_R^*} \rceil$, we know that the sub-matrix is always a fat matrix. When it is constructed following *Rule 1*, the L relay indexes in the matrix will be all different. Therefore, $\hat{\boldsymbol{C}}_b^{1,1}$ is a fat matrix composed of L statistically independent block matrices. This indicates that it is full row rank with probability one.

Type II: When $m > 1, j = 1$, the sub-matrix reduces to

$$\hat{\boldsymbol{C}}_b^{m,1} = \begin{bmatrix} \boldsymbol{P}_1 & & & | & \bar{\boldsymbol{E}}_{r_L^1} \\ \hline & \boldsymbol{P}_2 & & | & \bar{\boldsymbol{E}}_{r_L^1} \\ & & \ddots & | & \vdots \\ & & \boldsymbol{P}_m & | & \bar{\boldsymbol{E}}_{r_L^1} \end{bmatrix}. \quad \text{(A.2)}$$

When it is constructed following *Rule 1* and *Rule 2*, the $L-1$ relay indexes in each $\boldsymbol{P}_i$ are different, and the relay index sets in $\boldsymbol{P}_1, \ldots \boldsymbol{P}_m$ are not all the same. When it is constructed further following *Rule 3*, in the last column block of the last $m-1$ row blocks of $\hat{\boldsymbol{C}}_b^{m,1}$, there should be at least one "unerasable" block in each of the row blocks. As a result, the $m-1$ blocks of $\bar{\boldsymbol{E}}_{r_L^1}^T$ in the right lower corner of the sub-matrix should all be "unerasable". Moreover, the index r_L^1 will be different from all the relay indexes in $\boldsymbol{P}_1, \ldots \boldsymbol{P}_m$. This means $\bar{\boldsymbol{E}}_{r_L^1}^T$ is statistically independent of $\boldsymbol{P}_i, i = 1, \ldots, m$.

As defined in (22), $\boldsymbol{P}_i$ is of size $J \times (L-1)M_R^*$. Considering that the $L-1$ relay indexes in each of $\boldsymbol{P}_i$ are different and $J > (L-1)M_R^*$ according to the definition of L in Theorem 1, $\boldsymbol{P}_i$ is full column rank with probability one, i.e., $rank(\boldsymbol{P}_i) = (L-1)M_R^*$.

Using the property of the rank of matrix in [25], which is

$$rank([\boldsymbol{A}, \quad \boldsymbol{B}]) = rank(\boldsymbol{A}) + rank((I - \boldsymbol{A}\boldsymbol{A}^-)\boldsymbol{B}),$$

we can obtain the rank of $\hat{\boldsymbol{C}}_b^{m,1}$ as

$$rank\left(\hat{\boldsymbol{C}}_b^{m,1}\right) = \sum_{i=1}^{m} rank(\boldsymbol{P}_i) + rank\left(\boldsymbol{T}_m \bar{\boldsymbol{E}}_{r_L^1}\right), \quad \text{(A.3)}$$

where $\boldsymbol{T}_m = [\boldsymbol{P}_1^{\perp T} \quad \ldots \quad \boldsymbol{P}_m^{\perp T}]^T$, and $\boldsymbol{P}_i^{\perp} = \boldsymbol{I} - \boldsymbol{P}_i \boldsymbol{P}_i^-$ is the left null space of $\boldsymbol{P}_i$, therefore its rank is $rank(\boldsymbol{P}_i^{\perp}) = (B-1)D - (L-1)M_R^*$ [26].

According to random matrix theory, $rank(\boldsymbol{AB}) = \min\{rank(\boldsymbol{A}), rank(\boldsymbol{B})\}$ if random matrix $\boldsymbol{A}$ is statistically independent from a random matrix $\boldsymbol{B}$ [26]. Since $\bar{\boldsymbol{F}}_{r_L^1}^T$ is independent of $\boldsymbol{P}_i$, the second term in (A.3) is

$$rank\left(\boldsymbol{T}_m \bar{\boldsymbol{E}}_{r_L^1}\right) = \min\left\{rank(\boldsymbol{T}_m), rank\left(\bar{\boldsymbol{E}}_{r_L^1}\right)\right\}. \quad \text{(A.4)}$$

According to the property that $rank([\boldsymbol{A}^T, \quad \boldsymbol{B}^T]^T) \leq rank(\boldsymbol{A}) + rank(\boldsymbol{B})$ [25], the rank of matrix $\boldsymbol{T}_m$ is upper bounded as follows

$$rank(\boldsymbol{T}_m) \leq m\left[J - (L-1)M_R^*\right] \leq M_R^*, \quad \text{(A.5)}$$

where the last inequality is obtained from (23) with $j = 1$. Since $rank(\bar{\boldsymbol{E}}_{r_L^1}) = M_R^*$, we know from (A.4) that $rank(\boldsymbol{T}_m \bar{\boldsymbol{E}}_{r_L^1}) = rank(\boldsymbol{T}_m)$. Consequently, according to (A.3), we have

$$rank\left(\hat{\boldsymbol{C}}_b^{m,1}\right) = \sum_{i=1}^{m} rank(\boldsymbol{P}_i) + rank(\boldsymbol{T}_m). \quad \text{(A.6)}$$

In the following, we use *mathematical induction* to prove that for $1 \leq n \leq m$, we always have

$$rank(\boldsymbol{T}_n) = n[J - (L-1)M_R], \quad \text{(A.7)}$$

where $\boldsymbol{T}_n = [\boldsymbol{P}_{i_1}^{\perp T} \quad \ldots \quad \boldsymbol{P}_{i_n}^{\perp T}]^T, i_1, \ldots, i_n$ is arbitrarily re-ordered numbers of $1, \ldots, n$.

For $n = 1$, $\boldsymbol{T}_1 = \boldsymbol{P}_i$. The conclusion holds because $rank(\boldsymbol{P}_i^{\perp}) = J - (L-1)M_R^*$.

Suppose that for $1 < n < m$, (A.7) always holds. In this case, because $\bar{\boldsymbol{E}}_r$ is statistically independent of $\boldsymbol{P}_{i_1}, \ldots, \boldsymbol{P}_{i_n}$ and hence independent of $\boldsymbol{T}_n$, we have

$$\begin{aligned} rank(\boldsymbol{T}_n \bar{\boldsymbol{E}}_r) &= min\left\{rank(\boldsymbol{T}_n), rank\left(\bar{\boldsymbol{F}}_r^T\right)\right\} \\ &= n[J - (L-1)M_R^*]. \end{aligned} \quad \text{(A.8)}$$

Next, we prove that when $n = m$, (A.7) holds.

Using the property of matrix rank in [25], which is

$$rank\left([\boldsymbol{A}^T, \quad \boldsymbol{B}^T]^T\right) = rank(\boldsymbol{B}) + rank\left(\boldsymbol{A}(I - \boldsymbol{B}^- \boldsymbol{B})\right),$$

we can obtain the rank of $\boldsymbol{T}_m$ as

$$rank(\boldsymbol{T}_m) = rank\left(\boldsymbol{P}_m^{\perp}\right) + rank(\boldsymbol{T}_{m-1}\boldsymbol{P}_m), \quad \text{(A.9)}$$

where the second term is due to $\boldsymbol{P}_m = \boldsymbol{I} - (\boldsymbol{P}_m^{\perp})^-(\boldsymbol{P}_m^{\perp})$.

When the sub-matrix is constructed following *Rule* 2, the $L-1$ blocks in $\boldsymbol{P}_m$ are not all the same as those in $\boldsymbol{P}_i, i = 1, \ldots, m-1$. In other words, for each matrix $\boldsymbol{P}_i$, there exists at least one block $\bar{\boldsymbol{E}}_r$ in $\boldsymbol{P}_m$ that is different from all the $L-1$ blocks in $\boldsymbol{P}_i$.

If there exists one block in $\boldsymbol{P}_m$, say the block with subscript $r_1^m = r$, that is different from all the blocks in $\boldsymbol{P}_i, i = 1, \ldots, m-1$, the block $\bar{\boldsymbol{E}}_r$ will be statistically independent from all the blocks in $\boldsymbol{P}_1, \ldots, \boldsymbol{P}_{m-1}$. In this case, the second term in (A.9) is

$$\begin{aligned} rank(\boldsymbol{T}_{m-1}\boldsymbol{P}_m) &= rank(\boldsymbol{T}_{m-1}[\bar{\boldsymbol{E}}_r \quad \bar{\boldsymbol{P}}_m]) \\ &\geq rank(\boldsymbol{T}_{m-1}\bar{\boldsymbol{E}}_r), \end{aligned} \quad \text{(A.10)}$$

where $\bar{\boldsymbol{P}}_m$ is the matrix obtained from $\boldsymbol{P}_m$ by removing $\bar{\boldsymbol{E}}_r$.

Considering (A.8) and substituting (A.10) into (A.9), we can obtain that

$$rank(\boldsymbol{T}_m) \geq m\left[(B-1)D - (L-1)M_R^*\right]. \quad \text{(A.11)}$$

Further considering (A.5), we know that (A.7) holds.

If there does not exist such a block in $\boldsymbol{P}_m$, we can divide the matrices $\boldsymbol{P}_i, i = 1, \ldots, m-1$, into several groups by the following steps. First of all, the matrices $\boldsymbol{P}_i$ whose $L-1$ blocks are all different from the first block in $\boldsymbol{P}_m, \bar{\boldsymbol{E}}_{r_1^m}$, are in the first group and the number of the matrices $\boldsymbol{P}_i$ in this group is denoted as n_1. Then, among the rest matrixes whose $L-1$ blocks are all different from the second block in $\boldsymbol{P}_m, \bar{\boldsymbol{E}}_{r_2^m}$, are in the second group, and so on. In this way, $\bar{\boldsymbol{E}}_{r_j^m}$ in $\boldsymbol{P}_m$ is statistically independent from the blocks in matrices $\boldsymbol{P}_i$ of the jth group, and $\bar{\boldsymbol{E}}_{r_j^m}$ is the same as one of the blocks in matrices $\boldsymbol{P}_i$ of the next groups. In this case, the second term in (A.9) can be determined as

$$\begin{aligned} rank(\boldsymbol{T}_{m-1}\boldsymbol{P}_m) &= rank\left(\begin{bmatrix} \boldsymbol{T}_{n_1}\bar{\boldsymbol{E}}_{r_1^m} & * & * \\ \boldsymbol{0} & \ddots & * \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{T}_{n_{L-1}}\bar{\boldsymbol{E}}_{r_g^m} \end{bmatrix}\right) \\ &\geq rank\left(\boldsymbol{T}_{n_1}\bar{\boldsymbol{E}}_{r_1^m}\right) + \cdots rank\left(\boldsymbol{T}_{n_{L-1}}\bar{\boldsymbol{E}}_{r_{L-1}^m}\right), \end{aligned} \quad \text{(A.12)}$$

where $n_1, \ldots, n_{L-1}$ are respectively the numbers of the matrices $\boldsymbol{P}_i$ in each group, such that $n_1 + \cdots + n_{L-1} = m-1$, and $*$ are non-zero blocks.

Considering (A.8) and substituting (A.12) into (A.9), we can also obtain (A.11). Again with (A.5), we know that (A.7) holds when $n = m$. This completes the *mathematical induction*.

Finally, substituting (A.7) into (A.6), we can obtain the rank of $\hat{\boldsymbol{C}}_b^{m,1}$ as $rank(\hat{\boldsymbol{C}}_b^{m,1}) = mJ$, which is the number of rows in $\hat{\boldsymbol{C}}_b^{m,1}$. As a result, $\hat{\boldsymbol{C}}_b^{m,1}$ is full row rank.

Type III: When $j = m-1$, the sub-matrix reduces to

$$\hat{\boldsymbol{C}}_b^{m,m-1} = \left[\begin{array}{ccc|ccc} \boldsymbol{P}_1 & & & \bar{\boldsymbol{E}}_{r_L^1} & & \\ & \ddots & & & \ddots & \\ & & \boldsymbol{P}_{m-1} & & & \bar{\boldsymbol{E}}_{r_L^{m-1}} \\ \hline & & \boldsymbol{P}_m & \bar{\boldsymbol{E}}_{r_L^1} & \cdots & \bar{\boldsymbol{E}}_{r_L^{m-1}} \end{array}\right]. \quad (A.13)$$

When the sub-matrix is constructed following *Rule 3*, the blocks in the last $m-1$ column blocks of the last row block of $\hat{\boldsymbol{C}}_b^{m,m-1}$ should all be "unerasable". The relay indexes $r_L^1, \ldots, r_L^{m-1}$ are all different from the $L-1$ relay indexes in $\boldsymbol{P}_m$, and the relay index set of $\boldsymbol{P}_m$ is not the same as each relay index set of $\boldsymbol{P}_1, \ldots, \boldsymbol{P}_{m-1}$.

Divide each block $\bar{\boldsymbol{E}}_{r_L^i}$ as $\bar{\boldsymbol{E}}_{r_L^i} = [\,\bar{\boldsymbol{E}}_{r_L^i}^1 \quad \bar{\boldsymbol{E}}_{r_L^i}^2\,]$, where $\bar{\boldsymbol{E}}_{r_L^i}^1 \in \mathbb{C}^{J \times [J-(L-1)M_R^*]}$ and $\bar{\boldsymbol{E}}_{r_L^i}^2 \in \mathbb{C}^{J \times [LM_R^*-J]}, i = 1, \ldots, m-1$. Therefore, the matrix $[\,\boldsymbol{P}_i, \bar{\boldsymbol{E}}_{r_L^i}^1\,] \in \mathbb{C}^{J \times J}$ in the ith row block is a full rank square matrix. By taking matrix elementary transformation to the sub-matrix $\hat{\boldsymbol{C}}_b^{m,m-1}$ (first eliminating the block $\bar{\boldsymbol{E}}_{r_L^i}^2$ in each of the first $m-1$ row blocks using $[\,\boldsymbol{P}_i, \bar{\boldsymbol{E}}_{r_L^i}^1\,]$, and then eliminating the block $\bar{\boldsymbol{E}}_{r_L^i}^1$ in the last row block), we can correspondingly obtain the rank of $\hat{\boldsymbol{C}}_b^{m,m-1}$ as

$$\begin{aligned} rank\left(\hat{\boldsymbol{C}}_b^{m,m-1}\right) &= \sum_{i=1}^{m-1} rank\left(\left[\boldsymbol{P}_i, \bar{\boldsymbol{E}}_{r_L^i}^1\right]\right) \\ &\quad + rank\left(\left[\boldsymbol{P}_m, \bar{\boldsymbol{E}}_{r_L^1}^2 - \bar{\boldsymbol{E}}_{r_L^1}^1 \boldsymbol{Y}_1, \ldots, \right.\right. \\ &\qquad \left.\left. \bar{\boldsymbol{E}}_{r_L^{m-1}}^2 - \bar{\boldsymbol{E}}_{r_L^{m-1}}^1 \boldsymbol{Y}_{m-1}\right]\right), \end{aligned} \quad (A.14)$$

where $\boldsymbol{Y}_i \in \mathbb{C}^{[J-(L-1)M_R^*] \times [LM_R^*-J]}$ is a random matrix.

Using the property that $rank(\boldsymbol{A}, \boldsymbol{B}) = rank(\boldsymbol{A}) + rank(\boldsymbol{A}^{\perp}\boldsymbol{B})$, we further obtain that

$$\begin{aligned} rank\left(\hat{\boldsymbol{C}}_b^{m,m-1}\right) &= (m-1)J + (L-1)M_R^* \\ &\quad + rank\left(\boldsymbol{P}_m^{\perp}\left[\bar{\boldsymbol{E}}_{r_L^1}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_1\end{bmatrix}, \ldots, \bar{\boldsymbol{E}}_{r_L^{m-1}}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_{m-1}\end{bmatrix}\right]\right), \end{aligned} \quad (A.15)$$

where we used the fact that $rank([\boldsymbol{P}_i, \bar{\boldsymbol{E}}_{r_L^i}^1]) = J$ and $rank(\boldsymbol{P}_i) = (L-1)M_R^*$.

When the sub-matrix is constructed following *Rule 1*, the L relay indexes in the first $m-1$ row blocks are not all the same. Therefore, the blocks $\bar{\boldsymbol{E}}_{r_L^1}, \ldots, \bar{\boldsymbol{E}}_{r_L^{m-1}}$ can be set to be different. With the construction *Rule 3*, the relay indexes $r_L^1, \ldots, r_L^{m-1}$ are all different from the $L-1$ relay indexes in $\boldsymbol{P}_m$. This suggests that $\bar{\boldsymbol{E}}_{r_L^i}$ is statistically independent from $\boldsymbol{P}_m$. As a result, the matrix

$$\left[\bar{\boldsymbol{E}}_{r_L^1}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_1\end{bmatrix}, \ldots, \bar{\boldsymbol{E}}_{r_L^{m-1}}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_{m-1}\end{bmatrix}\right] \quad (A.16)$$

is a full rank matrix of size $J \times (m-1)[LM_R^* - J]$ and independent from $\boldsymbol{P}_m$. Then, the rank of the third term in (A.15) is

$$\begin{aligned} &rank\left(\boldsymbol{P}_m^{\perp}\left[\bar{\boldsymbol{E}}_{r_L^1}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_1\end{bmatrix}, \ldots, \bar{\boldsymbol{E}}_{r_L^{m-1}}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_{m-1}\end{bmatrix}\right]\right) \\ &\qquad = min\left\{J - (L-1)M_R^*, (m-1)[LM_R^* - J]\right\}. \end{aligned}$$

Since $\hat{\boldsymbol{C}}_b^{m,m-1}$ is a fat matrix, we can obtain that $mJ \leq m(L-1)M_R^* + (m-1)M_R^*$, or equivalently $J - (L-1)M_R^* \leq (m-1)[LM_R^* - J]$. We can correspondingly obtain that $rank(\boldsymbol{P}_m^{\perp}[\bar{\boldsymbol{E}}_{r_L^1}[\begin{smallmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_1\end{smallmatrix}], \ldots, \bar{\boldsymbol{E}}_{r_L^{m-1}}[\begin{smallmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_{m-1}\end{smallmatrix}]]) = J - (L-1)M_R^*$. Upon substituting into (A.15), we have

$$rank\left(\hat{\boldsymbol{C}}_b^{m,m-1}\right) = mJ. \quad (A.17)$$

This indicates that the sub-matrix $\hat{\boldsymbol{C}}_b^{m,m-1}$ is full row rank.

Type IV: When m and j satisfy conditions (11b) and (11c), the sub-matrix is in the general form as in (22).

When the sub-matrix is constructed following *Rule 1*, all the N relay indexes in the first j row blocks of $\hat{\boldsymbol{C}}_b^{m,j}$ should be different and they are not all the same as each other. When following *Rule 2*, the $L-1$ relay indexes from its $(j+1)$th row block to the mth block are not all the same. When following *Rule 3*, there is at least one relay index in $r_L^1, \ldots, r_L^j$ that is different from all the $L-1$ relay indexes in $\boldsymbol{P}_{j+1}, \ldots, \boldsymbol{P}_m$.

Similar to the derivation for (A.14), we can obtain the rank of $\hat{\boldsymbol{C}}_b^{m,j}$ as

$$\begin{aligned} rank\left(\hat{\boldsymbol{C}}_b^{m,j}\right) &= \sum_{i=1}^{j} rank\left(\left[\boldsymbol{P}_i, \bar{\boldsymbol{F}}_{r_L^i}^{T1}\right]\right) + \sum_{i=j+1}^{m} rank\left(\boldsymbol{P}_i\right) \\ &\quad + rank\left(\underbrace{\begin{bmatrix}\boldsymbol{P}_{j+1}^{\perp}\\ \vdots \\ \boldsymbol{P}_m^{\perp}\end{bmatrix}}_{\boldsymbol{T}_{m-j}}\left[\bar{\boldsymbol{E}}_{r_L^1}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_1\end{bmatrix}, \ldots, \bar{\boldsymbol{E}}_{r_L^j}\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_j\end{bmatrix}\right]\right). \end{aligned} \quad (A.18)$$

Using the same way for the matrix in (A.16), $[\bar{\boldsymbol{E}}_{r_L^1}[\begin{smallmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_1\end{smallmatrix}], \ldots, \bar{\boldsymbol{E}}_{r_L^j}[\begin{smallmatrix}\boldsymbol{I}\\ \boldsymbol{Y}_j\end{smallmatrix}]]$ can also be constructed as a full rank matrix. Then, the third term of (A.18) equals to the rank of $\boldsymbol{T}_{m-j}$.

Using the same fact to derive (A.15) and using (A.7) to determine the rank of the matrix $\boldsymbol{T}_{m-j}$, from (A.18) we have

$$\begin{aligned} rank\left(\hat{\boldsymbol{C}}_b^{m,j}\right) &= jJ + (m-j)(L-1)M_R^* \\ &\quad + (m-j)[J - (L-1)M_R^*] = mJ. \end{aligned} \quad (A.19)$$

This completes the proof. ∎

APPENDIX B
PROOF OF LEMMA 2

Proof: Remind that matrix $\hat{\boldsymbol{C}}_b$ in (20) has D row blocks and R_B column blocks. Condition (11a) ensures that $\hat{\boldsymbol{C}}_b$ is a fat matrix, which is necessary to ensure it as full row rank.

An immediate way to construct $\hat{\boldsymbol{C}}_b$ is to set it as a type IV matrix shown in (22), where $m = D, j = n_2$. However, such a simple construction does not guarantee the matrix to be full rank. To construct a full rank matrix $\hat{\boldsymbol{C}}_b$, we first construct it as a block diagonal matrix shown in (25) by setting corresponding scalars "g" as "0" and "1", which is composed of multiple sub-matrices $\hat{\boldsymbol{C}}_b^{m,j}$. Then, we construct the sub-matrices that are full rank.

From Lemma 1 we know that if $\hat{\boldsymbol{C}}_b^{m,j}$ is constructed following the three rules, then the sub-matrix is full rank. After Lemma 1 we have shown that if m, j satisfy (24a) and (24b), $\hat{\boldsymbol{C}}_b^{m,j}$ is full rank. If we can further show that the conditions in (24a) and (24b) have the same expressions as those in (11b) and (11c), then we can prove this lemma.

Suppose that the matrix $\hat{\boldsymbol{C}}_b$ are composed of $l_{m,j}$ sub-matrices $\hat{\boldsymbol{C}}_b^{m,j}$, where $l_{m,j} \geq 0, m \leq D, j \leq n_2$ and $\frac{m}{j} \leq \frac{M_R^*}{n_1}$ as shown in (23).

Define a parameter set $\Omega = \{(m, j, l_{m,j}) \mid l_{m,j} \geq 0, m \leq D, j \leq n_2, \frac{m}{j} \leq \frac{M_R^*}{n_1}\}$. If we can find a subset of Ω^0 that satisfies the following conditions

$$\sum_{(m,j,l_{m,j})\in\Omega^0} m l_{m,j} = D, \sum_{(m,j,l_{m,j})\in\Omega^0} j l_{m,j} \leq n_2, \quad \text{(B.2)}$$

i.e., the multiple sub-matrices can compose the block diagonal matrix shown in (25), and all the sub-matrices are full rank, then Lemma 2 is proved.

Whether we can find a subset Ω^0 satisfying (B.2) depends on the relationships between B, D, M_R^*, n_1 and n_2. From condition (11a) and the definition of n_1 and n_2, we can obtain

$$\frac{D}{n_2} \leq \frac{M_R^*}{n_1}. \quad \text{(B.3)}$$

In the following, we find the subset considering different relationships of the parameters.

From the relationship $n_1 = J - (L-1)M_R^* \leq M_R$, we know that $\frac{M_R^*}{n_1} \geq 1$. In the sequel, we find corresponding subsets Ω^0 according to whether $\frac{M_R^*}{n_1} > 2$ or $1 \leq \frac{M_R^*}{n_1} < 2$ and the relationship of $\frac{M_R^*}{n_1}$ and $\frac{D}{n_2}$.

Case 1: $\frac{M_R^*}{n_1} \geq 2$.

Case 1.1: $\frac{D}{n_2} \leq 1$ and $\frac{M_R^*}{n_1} \geq 2$.

In this case, a subset Ω^0 satisfying (B.2) is

$$\Omega^0 = \{(m = 1, j = 1, l_{m,j} = D)\}, \quad \text{(B.4)}$$

which means that the matrix $\hat{\boldsymbol{C}}_b$ can be composed of D sub-matrices of $\hat{\boldsymbol{C}}_b^{1,1}$, i.e., type I matrices.

From (B.4), we know that in this case, $j = 1, m - j + 1 = 1$. Then, conditions (24a) and (24b) as well as conditions (11b) and (11c) become

$$\binom{N_R}{L} \geq 1 \text{ and } \binom{N_R - 1}{L-1} \geq 1, \quad \text{(B.5)}$$

which are automatically satisfied because $N_R \geq L$.

This means that when the conditions in Theorem 1 are satisfied, the matrix $\hat{\boldsymbol{C}}_b$ can be composed as a block diagonal matrix whose diagonal matrices are full row rank type I matrices.

Case 1.2: $1 < \frac{D}{n_2} \leq \lfloor \frac{M_R^*}{n_1} \rfloor \leq \frac{M_R^*}{n_1}$ and $\frac{M_R^*}{n_1} \geq 2$.

In this case, the values of m and j should be judiciously chosen. We rewrite (B.2) as

$$m_1 l_{m_1,j_1} + m_2 l_{m_2,j_2} = D,$$
$$j_1 l_{m_1,j_1} + j_2 l_{m_2,j_2} = n_2, \quad \text{(B.6)}$$

where the parameters m_1, m_2, j_1 and j_2 should satisfy $\frac{m_1}{j_1} \leq \frac{D}{n_2} \leq \frac{m_2}{j_2} \leq \frac{M_R^*}{n_1}$.

Because $\frac{D}{n_2} \leq \lfloor \frac{M_R^*}{n_1} \rfloor$, we can obtain that $\lfloor \frac{D}{n_2} \rfloor \leq \frac{D}{n_2} \leq \lceil \frac{D}{n_2} \rceil \leq \frac{M_R^*}{n_1}$. Therefore, we can find a subset Ω^0 satisfying (B.6) as follows

$$m_1 = \left\lfloor \frac{D}{n_2} \right\rfloor, j_1 = 1, l_{m_1,j_1} = \left\lceil \frac{D}{n_2} \right\rceil n_2 - D,$$
$$m_2 = \left\lceil \frac{D}{n_2} \right\rceil, j_2 = 1, l_{m_2,j_2} = D - \left\lfloor \frac{D}{n_2} \right\rfloor n_2. \quad \text{(B.7)}$$

From (B.7), we know that in this case, $j_1 = j_2 = 1, m_1 - j_1 + 1 = \lfloor \frac{D}{n_2} \rfloor, m_2 - j_2 + 1 = \lceil \frac{D}{n_2} \rceil$. By setting $j = max\{j_1, j_2\}$ and $m - j + 1 = max\{m_1 - j_1 + 1, m_2 - j_2 + 1\}$, then the two conditions in (24a) and (24b) become

$$\binom{N_R}{L} \geq 1,$$
$$\binom{N_R - 1}{L-1} \geq \left\lceil \frac{D}{n_2} \right\rceil. \quad \text{(B.8)}$$

On the other hand, the parameter c in conditions (11b) and (11c) has different forms according to whether $\frac{D}{n_2} > 2$ or $1 < \frac{D}{n_2} \leq 2$.

When $\frac{D}{n_2} > 2$, we can obtain that $1 \leq \frac{D}{D-n_2} < 2$, then $c_1 = \lfloor \frac{D}{n_2} \rfloor$. Because $\frac{M_R^*}{n_1} \geq 2$, we can obtain $c_2 = \frac{M_R^*}{n_1}$. According to the definition of c in Theorem 1, we can obtain that when $\lceil \frac{D}{n_2} \rceil \neq \frac{M_R^*}{n_1}$, then $\frac{M_R^*}{n_1} - \lceil \frac{D}{n_2} \rceil \geq 1$, which result in $\lceil \frac{1}{c} \rceil = 1$. When $\lceil \frac{D}{n_2} \rceil = \frac{M_R^*}{n_1}, c = 1$. As a result, we always have $\lceil \frac{1}{c} \rceil = 1$.

When $\frac{D}{n_2} \leq 2$, we can obtain that $\frac{D}{D-n_2} \geq 2$, then $c_1 = \lceil \frac{D}{D-n_2} \rceil$, while we still have $c_2 = \frac{M_R^*}{n_1}$. Again from the definition of c, we can obtain that $c \geq 1$. As a result, we always have $\lceil \frac{1}{c} \rceil = 1$. Then conditions (11b) and (11c) become

$$\binom{N_R}{L} \geq \left\lfloor \frac{D}{D - n_2} \right\rfloor \geq 1,$$
$$\binom{N_R - 1}{L-1} \geq \left\lceil \frac{D}{n_2} \right\rceil. \quad \text{(B.9)}$$

Further considering (B.8), we can see that when conditions (11b) and (11c) are satisfied, the sub-matrices $\hat{\boldsymbol{C}}_b^{m_1,j_1}$ and $\hat{\boldsymbol{C}}_b^{m_2,j_2}$ will be full row rank, which are type II matrices.

Consequently, when the conditions in Theorem 1 are satisfied, the matrix $\hat{\boldsymbol{C}}_b$ can be composed as a block diagonal matrix whose diagonal matrices are full row rank type II matrices.

Case 1.3: $2 \le \lfloor \frac{M_R^*}{n_1} \rfloor < \frac{D}{n_2} \le \frac{M_R^*}{n_1}$.

In this case, we know that $\lfloor \frac{D}{n_2} \rfloor < \frac{D}{n_2} \le \frac{M_R^*}{n_1}$. We also rewrite (B.2) as (B.6). Then, we need to find a subset $\Omega^0 = \{(m_1, j_1, l_{m_1,j_1}), (m_2, j_2, l_{m_2,j_2})\}$ satisfying (B.6).

By setting $m_1 = \lfloor \frac{D}{n_2} \rfloor, j_1 = 1$, (B.6) becomes

$$\left\lfloor \frac{D}{n_2} \right\rfloor l_{m_1,j_1} + m_2 l_{m_2,j_2} = D, \tag{B.9a}$$

$$l_{m_1,j_1} + j_2 l_{m_2,j_2} = n_2. \tag{B.9b}$$

By multiplying $\lfloor \frac{D}{n_2} \rfloor$ to (B.9b) and subtracting it from (B.9a), we can obtain that

$$\left(m_2 - \left\lfloor \frac{D}{n_2} \right\rfloor j_2\right) l_{m_2,j_2} = D - \left\lfloor \frac{D}{n_2} \right\rfloor n_2. \tag{B.10}$$

One solution of the subset Ω^0 satisfying (B.10) can be determined as follows: we find m and j from $m_2 - \lfloor \frac{D}{n_2} \rfloor j_2 = 1$, and $l_{m,j}$ equals the right-hand side of (B.10). According to the relationship that $\frac{D}{n_2} \le \frac{m_2}{j_2} \le \frac{M_R^*}{n_1}$, we can find a subset Ω^0 as

$$m_1 = \left\lfloor \frac{D}{n_2} \right\rfloor, j_1 = 1,$$
$$m_2 = \left\lfloor \frac{D}{n_2} \right\rfloor \left\lceil \frac{1}{\frac{M_R^*}{n_1} - \left\lfloor \frac{D}{n_2} \right\rfloor} \right\rceil + 1, j_2 = \left\lceil \frac{1}{\frac{M_R^*}{n_1} - \left\lfloor \frac{D}{n_2} \right\rfloor} \right\rceil,$$

from which we can obtain that $j_2 > j_1$ and $m_2 - j_2 + 1 \ge m_1 - j_1 + 1$ because $\frac{M_R^*}{n_1} - \lfloor \frac{D}{n_2} \rfloor = \frac{M_R^*}{n_1} - \lfloor \frac{M_R^*}{n_1} \rfloor < 1$. Then the two condition in (24a) and (24b) becomes

$$\binom{N_R}{L} \ge \left\lceil \frac{1}{\frac{M_R^*}{n_1} - \left\lfloor \frac{D}{n_2} \right\rfloor} \right\rceil,$$
$$\binom{N_R - 1}{L - 1} \ge \left(\left\lfloor \frac{D}{n_2} \right\rfloor - 1\right) \left\lceil \frac{1}{\frac{M_R^*}{n_1} - \left\lfloor \frac{D}{n_2} \right\rfloor} \right\rceil + 2. \tag{B.11}$$

On the other hand, because $2 \le \lfloor \frac{M_R^*}{n_1} \rfloor < \frac{D}{n_2} \le \frac{M_R^*}{n_1}$, the parameters c_1 and c_2 are respectively determined as $c_1 = \lceil \frac{D}{n_2} \rceil$ and $c_2 = \frac{M_R^*}{n_1}$. We can also obtain that $\lfloor \frac{D}{D-n_2} \rfloor = 1$. Then conditions (11b) and (11c) become

$$\binom{N_R}{L} \ge \left\lceil \frac{1}{\frac{M_R^*}{n_1} - \left\lfloor \frac{D}{n_2} \right\rfloor} \right\rceil,$$
$$\binom{N_R - 1}{L - 1} \ge \left\lfloor \frac{D}{n_2} \right\rfloor \left\lceil \frac{1}{\frac{M_R^*}{n_1} - \left\lfloor \frac{D}{n_2} \right\rfloor} \right\rceil + 1$$
$$\overset{(a)}{\ge} \left(\left\lfloor \frac{D}{n_2} \right\rfloor - 1\right) \left\lceil \frac{1}{\frac{M_R^*}{n_1} - \left\lfloor \frac{D}{n_2} \right\rfloor} \right\rceil + 2. \tag{B.12}$$

where (a) is obtained because $\frac{M_R^*}{n_1} - \lfloor \frac{D}{n_2} \rfloor < 1$.

We can see that the conditions in (B.12) are the same as those in (B.11). This indicates that when conditions (11b) and (11c) are satisfied, the sub-matrices $\hat{\boldsymbol{C}}_b^{m_1,j_1}$ and $\hat{\boldsymbol{C}}_b^{m_2,j_2}$ will be full row rank. They are respectively a type II and type IV matrices.

Case 2: $1 \le \frac{M_R^*}{n_1} < 2$ (or equivalently $\frac{M_R^*}{M_R^* - n_1} > 2$).

Case 2.1: $\frac{D}{n_2} \le 1 < \frac{M_R^*}{n_1} \le 2$.

In this case, the subset Ω^0 satisfying (B.2) is the same as in *Case 1.1*. When (24a) and (24b) are satisfied, the two conditions in (11b) and (11c) are also satisfied due to the same reason as in *Case 1.1*.

Cases 2.2 and 2.3 come from (B.3): $1 < \frac{D}{n_2} \le \frac{M_R^*}{n_1}$, which results in $2 < \frac{M_R^*}{M_R^* - n_1} < \frac{D}{D - n_2}$.

Case 2.2: $2 < \frac{M_R^*}{M_R^* - n_1} \le \lceil \frac{M_R^*}{M_R^* - n_1} \rceil \le \frac{D}{D - n_2}$ and $1 \le \frac{M_R^*}{n_1} < 2$.

In this case, we can rewrite (B.2) as

$$m_1 l_{m_1,j_1} + m_2 l_{m_2,j_2} = D,$$
$$(m_1 - j_1) l_{m_1,j_1} + (m_2 - j_2) l_{m_2,j_2} = D - n_2, \tag{B.13}$$

where $\frac{M_R^*}{M_R^* - n_1} \le \frac{m_1}{m_1 - j_1} \le \frac{D}{D - n_2} \le \frac{m_2}{m_2 - j_2}$.

Therefore, we can find a subset Ω^0 satisfying (B.13) as follows

$$m_1 = \left\lfloor \frac{D}{D - n_2} \right\rfloor, m_1 - j_1 = 1,$$
$$m_2 = \left\lceil \frac{D}{D - n_2} \right\rceil, m_2 - j_2 = 1, \tag{B.14}$$

from which we can obtain that $j_1 = \lfloor \frac{D}{D-n_2} \rfloor - 1, j_2 = \lfloor \frac{D}{D-n_2} \rfloor$, and $m_1 - j_1 + 1 = m_2 - j_2 + 1 = 2$. Then the two conditions in (24a) and (24b) become

$$\binom{N_R}{L} \ge \left\lfloor \frac{D}{D - n_2} \right\rfloor,$$
$$\binom{N_R - 1}{L - 1} \ge 2. \tag{B.15}$$

In this case, because $1 \le \frac{D}{n_2} \le \frac{M_R^*}{n_1} < 2$, we have $c_1 = \lfloor \frac{D}{D-n_2} \rfloor$ and $c_2 = \frac{M_R^*}{M_R^* - n_1}$. Moreover, we have $\lfloor \frac{D}{n_2} \rfloor = 1$. According to the definition of c, we know that when $c_1 \ne c_2, c_2 - c_1 \ge 1$, therefore, $\lceil \frac{1}{c} \rceil = 1$. Then the conditions in (11b) and (11c) become exactly the same as (B.15).

This indicates that when conditions (11b) and (11c) satisfy, the sub-matrices $\hat{\boldsymbol{C}}_b^{m_1,j_1}$ and $\hat{\boldsymbol{C}}_b^{m_2,j_2}$ will be full row rank. They are all type III matrices according to (B.14).

Case 2.3: $2 < \frac{M_R^*}{M_R^* - n_1} \le \frac{D}{D - n_2} \le \lceil \frac{M_R^*}{M_R^* - n_1} \rceil$ and $1 \le \frac{M_R^*}{n_1} < 2$.

In this case, we first set $m_1 = \lceil \frac{D}{D-n_2} \rceil, m_1 - j_1 = 1$, we can rewrite (B.2) as

$$\left\lceil \frac{D}{D - n_2} \right\rceil l_{m_1,j_1} + m_2 l_{m_2,j_2} = D, \tag{B.15a}$$
$$l_{m_1,j_1} + (m_2 - j_2) l_{m_2,j_2} = D - n_2. \tag{B.15b}$$

By multiplying $\lceil \frac{D}{D-n_2} \rceil$ to (B.15b) and subtracting it from (B.15a), we can obtain that

$$\left[\left\lceil \frac{D}{D-n_2} \right\rceil (m_2 - j_2) - m_2 \right] l_{m_2,j_2} = \left\lceil \frac{D}{D-n_2} \right\rceil (D - n_2) - D. \quad \text{(B.16)}$$

One solution of the subset Ω^0 satisfying (B.16) can be determined as follows: we find m and j from $\lceil \frac{D}{D-n_2} \rceil (m_2 - j_2) - m_2 = 1$, and $l_{m,j}$ equals the right-hand side of (B.16). According to the relationship that $\frac{M_R^*}{M_R^* - n_1} \leq \frac{m_2}{m_2 - j_2} \leq \frac{D}{D-n_2}$, we can find a subset Ω^0 as

$$\begin{aligned} m_1 &= \left\lceil \frac{D}{D-n_2} \right\rceil, m_1 - j_1 = 1, \\ m_2 &= \left\lceil \frac{D}{D-n_2} \right\rceil \left\lceil \frac{1}{\left\lceil \frac{D}{D-n_2} \right\rceil - \frac{M_R^*}{M_R^* - n_1}} \right\rceil - 1, \\ m_2 - j_2 &= \left\lceil \frac{1}{\left\lceil \frac{D}{D-n_2} \right\rceil - \frac{M_R^*}{M_R^* - n_1}} \right\rceil, \end{aligned} \quad \text{(B.17)}$$

from which we can obtain $j_1 = \lfloor \frac{D}{D-n_2} \rfloor, j_2 = \lfloor \frac{D}{D-n_2} \rfloor \lceil \frac{1}{\lceil \frac{D}{D-n_2} \rceil - \frac{M_R^*}{M_R^* - n_1}} \rceil - 1 > j_1$. Then the two conditions in (24a) and (24b) become

$$\begin{aligned} \binom{N_R}{L} &\geq \left\lfloor \frac{D}{D-n_2} \right\rfloor \left\lceil \frac{1}{\frac{D}{D-n_2} - \left\lceil \frac{M_R^*}{M_R^* - n_1} \right\rceil} \right\rceil - 1, \\ \binom{N_R - 1}{L - 1} &\geq \left\lceil \frac{1}{\left\lceil \frac{D}{D-n_2} \right\rceil - \frac{M_R^*}{M_R^* - n_1}} \right\rceil + 1. \end{aligned} \quad \text{(B.18)}$$

On the other hand, we also have $c_1 = \lfloor \frac{D}{D-n_2} \rfloor, c_2 = \frac{M_R^*}{M_R^* - n_1}$ and $\lfloor \frac{D}{n_2} \rfloor = 1$. Then the conditions in (11b) and (11c) become

$$\begin{aligned} \binom{N_R}{L} &\geq \left\lfloor \frac{D}{D-n_2} \right\rfloor \left\lceil \frac{1}{\frac{D}{D-n_2} - \left\lceil \frac{M_R^*}{M_R^* - n_1} \right\rceil} \right\rceil, \\ \binom{N_R - 1}{L - 1} &\geq \left\lceil \frac{1}{\left\lceil \frac{D}{D-n_2} \right\rceil - \frac{M_R^*}{M_R^* - n_1}} \right\rceil + 1. \end{aligned} \quad \text{(B.19)}$$

When these conditions satisfy, those in (B.18) will satisfy.

This indicates that when conditions (11b) and (11c) satisfy, the sub-matrices $\hat{\boldsymbol{C}}_b^{m_1,j_1}$ and $\hat{\boldsymbol{C}}_b^{m_2,j_2}$ are full row rank.

Summarizing all the cases, we now have shown that when the conditions in Theorem 1 are satisfied, the matrix $\hat{\boldsymbol{C}}_b$ can be constructed as a full row rank block diagonal matrix. When $\frac{D}{n_2} \leq 1$ as in Case 1.1 and Case 2.1, the constructed $\hat{\boldsymbol{C}}_b$ is only composed of full row rank type I sub-matrices. Otherwise, the constructed matrices are composed of type II, type III and type IV sub-matrices. This completes the proof. ■

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