THE EFFECTS OF NOISE, SPARSITY AND PHASE ON PSEUDO-RANDOM TIME-SPACE MODULATION SAR PERFORMANCE

Ying Liu, Ze Yu, Wenjiao Chen, Jindong Yu, and Jiwen Geng

School of Electronics and Information Engineering, Beihang University, Beijing 100191, China

ABSTRACT

SAR based on compressed sensing (CS) greatly reduces the amount of data. The pseudo-random space-time modulation technology could alleviate the constraint on the type of the observed scene. This paper provides a survey on the effects of noise, sparsity, and phase on the modulation technology performance. Selection of a suitable algorithm is necessary to achieve this goal. $l_1 - norm$ algorithm performs the best of the three algorithms, including greedy algorithm, and Bayesian algorithm. The experiment results show that the performance of the pseudo-random space-time modulation SAR is improved. With the increase of the noise and the decrease of the sparsity, the performance improvement with modulation is more and more limited. With the decrease of phase density, the performance obtained by modulation decreases continuously. When the amplitude variation range exceeds $[0,\pi]$, the improvements are similar.

Index Terms— pseudo-random modulation, reconstruction algorithm, noise, sparsity, phase

1. INTRODUCTION

High-resolution and wide-swath request SAR system based on the Nyquist sampling theorem to acquire and process more and more data. A certain degree of redundancy exists in the echo, which means radar data is compressible [1]. CS can greatly reduce the amount of data. If the measurement matrix satisfies condition of restricted isometry property (RIP), original sparse signal can be recovered from a small set of linear nonadaptive measurements, much smaller in size than required by the Nyquist sampling theorem [2].

An alternative approach based on CS for radar imaging is proposed. This approach regards azimuth focusing by taking only a fraction of the temporal signal sequence [3]. However, the CS-SAR proposed above is limited in the type of the observed scene and can only reconstruct the sparse scene. In order to alleviate this limitation, a pseudo-random space-time modulation SAR has been proposed. This technology is phase modulation which generates random phase along azimuth dimension [4].

To explore the applicability of this technique, we study the effects of noise, sparsity, phase density and amplitude on pseudo-random space-time modulation SAR technology. Airborne SAR and spaceborne SAR have different SNR for different scenes. It is of great significance to find out the sparsity of observation scene which this technique is more suitable for in practice. The existing antenna technology can not completely meet the requirements of the modulation technology. This survey provides a reference for designing a SAR system to observe different scenes.In order to reduce the recovery error, the performance of different categories reconstruction algorithm is also compared.

This paper is organized as follows. Section 2 describes the echo model in detail. Section 3 presents the comparison of three CS reconstruction algorithms. Simulations results and analysis of the effects of noise, sparsity and phase on modulation performance are represented in section 4.

2. ECHO SIGNAL MODEL

The principle of synthetic aperture is that the coherent information recorded at the different positions is used to synthesize a larger antenna to obtain higher azimuth resolution. The antennas in traditional SAR systems mainly include parabolic antenna and phased-array antenna. The Lincoln Laboratory in the Massachusetts Institute of Technology (MIT) proposed compressive reflector antenna (CRA) which can generate spatially and temporally variable random phase [6]. Pseudo-random space-time modulation can be achieved by the CRA. In the pseudo-random space-time modulation, the phase is random. “Space” means that different imaging areas have different random phases, and “time” means that different sampling moments have different random phases.

After range compression and RCMC, the echo signal model can be expressed as

$$\begin{aligned} p_{rcm}(\tau,\eta) = \sum_{x_i y_i} \Bigg\{ & \sigma_i W_i(\sigma,\eta) T_r \sin c\left\{ K_r T_r \left[\tau - \frac{2R_i(\eta_{ci})}{c} \right] \right\} \\ & \cdot \exp\left\{ -\frac{j4\pi R_i(\eta-\eta_{ci})}{\lambda} \right\} \cdot \exp\left\{ -j\varphi_i(\eta) \right\} \Bigg\} \\ & + n(\tau,\eta) \end{aligned} \tag{1}$$

where $\varphi_i(\eta)$ represents the phase generated by the antenna corresponding to the point target (x_i, y_i). The phase is linear in the traditional antenna, while the phase is random in CRA. $n(\tau,\eta)$ represents the additive noise in the data.

For a range bin cell corresponding to the range sampling moment τ_0, Eq.(1) can be expressed as

$$p_{N\times1} = D_{N\times M}\sigma_{M\times1} + n_{N\times1} \tag{2}$$

$$\begin{bmatrix} s_c(\tau_0,\eta_1) \\ s_c(\tau_0,\eta_2) \\ \vdots \\ s_c(\tau_0,\eta_N) \end{bmatrix} = \begin{bmatrix} D_1(\tau_0,\eta_1) & D_2(\tau_0,\eta_1) & \cdots & D_M(\tau_0,\eta_1) \\ D_1(\tau_0,\eta_2) & D_2(\tau_0,\eta_2) & \cdots & D_M(\tau_0,\eta_2) \\ \vdots & \vdots & \ddots & \vdots \\ D_1(\tau_0,\eta_N) & D_2(\tau_0,\eta_N) & \cdots & D_M(\tau_0,\eta_N) \end{bmatrix} \times \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_M \end{bmatrix} + \begin{bmatrix} n(\tau_0,\eta_1) \\ n(\tau_0,\eta_2) \\ \vdots \\ n(\tau_0,\eta_N) \end{bmatrix}$$

$$D_i(\sigma_0,\eta_j) = \begin{cases} 0 & \eta_j - \eta_{ci} > \dfrac{T_a}{2} \\ W_i(\sigma_0,\eta_j)\cdot T_r \\ \cdot \sin c\left\{K_r T_r\left(\tau_0 - \dfrac{2R_i(\eta-\eta_{ci})}{c}\right)\right\} & \eta_j - \eta_{ci} \le \dfrac{T_a}{2} \\ \cdot \exp\left\{\dfrac{-j4\pi R_i(\eta_j-\eta_{ci})}{\lambda}\right\} \\ \cdot \exp\{-j\varphi_i(\eta_j)\} \end{cases}$$

$$0 \le i \le M, 0 \le j \le N$$

The RIP property is a sufficient and necessary condition for accurate reconstruction of the original signal in CS. However, it is difficult to prove whether the matrix satisfies the RIP condition, so this property is not used extensively in practical applications. In general, the performance of the reconstruction matrix is evaluated through mutual coherence coefficient, which reflects the maximum similarity between any two different columns. It can be expressed as

$$u = \max_{i\neq i} \frac{\langle D_i, D_j \rangle}{\|D_i\|_2 \bullet \|D_j\|_2} \tag{3}$$

where D_i and D_j denote the ith and jth column of the matrix D respectively. The matrix with large mutual coherence coefficient is not good for the reconstruction of the original signal.

Table.1 Simulation parameters

Parameters	Value
Orbit Hight(km)	600
Velocity (m/s)	7100
Resolution(m)	2.658
Wavelength(m)	0.03
Squint Angle (°)	0

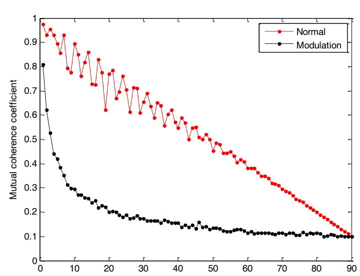


Fig.1 Illustration of mutual coherence coefficients

In traditional SAR, the azimuth observation matrix is determined by the Doppler movement between the radar and the observed scene. After pseudo-random space-time modulation, the mutual coherence coefficient decreases and the randomness increases, which is more conducive to signal recovery. The simulation shown in Fig. 1 also verifies this, and the corresponding simulation parameters are given in Table.1.

3. COMPRESSED SENSING ALGORITHM

3.1 Compressed sensing reconstruction algorithm

The reconstruction equation without subscript is written as:

$$p = D\sigma + n \tag{4}$$

The most fundamental method to reconstruct the azimuth signal is to solve the following $l_0 - norm$ equation:

$$\min_{\sigma} \|\sigma\|_0 \quad s.t. \ \|D\sigma - p\|_2 \le \varepsilon \tag{5}$$

Many sparse recovery algorithms have been proposed to solve this problem. The algorithms are mainly divided into three categories: convex relaxation algorithm, greedy algorithm, and Bayesian algorithm.

The first category, convex relaxation algorithm, solves the sparse signal recovery problem by transforming non-convex problem into convex optimization problem. Example of this technique includes Basis Pursuit De-Noising (BPDN). $l_0 - norm$ and $l_1 - norm$ are equivalent under certain conditions. Eq.(5) can be rewritten as

$$\min_{\sigma} \|\sigma\|_1 \quad s.t. \ \|D\sigma - p\|_2 \le \varepsilon \tag{6}$$

Eq.(6) can also be written as

$$\min_{\sigma} \ \|D\sigma - p\|_2 + \lambda\|\sigma\|_1 \tag{7}$$

where λ is the regularization parameter, which is often an experience value.

The second category, greedy algorithm, recovers the sparse signal through an iterative process. Example of this technique includes Orthogonal Matching Pursuit (OMP).

The third category, Bayesian Compressive Sensing (BCS), solves the sparse recovery problem by taking into account a prior knowledge of the sparse signal distribution.

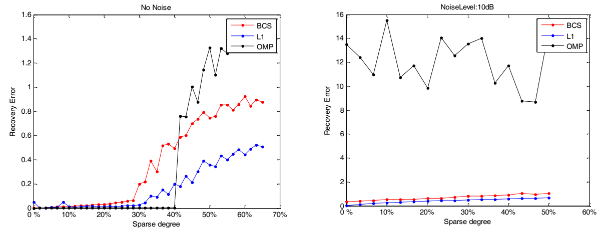
3.2 Experimental results

In this work, we consider a one-dimensional scene of length N=1501 that contain K point targets randomly chosen with random backscatter amplitude and phase. K is variable, which determines the sparsity varies from 1% to 50%. Table.1 shows the simulation parameters. The modulation phases uniformly distributed in $[0, 2\pi]$. The additional noise obeys a Gaussian distribution. The sampling rate was reduced to 1/4. Three sparse recovery algorithms are simulated under the conditions of 0 noise and 10dB SNR.

In order to compare the performance of sparse recovery algorithms, we use recovery error to evaluate the error between the original sparse signal and the recovered one. We used the following equation to calculate this error:

$$Err = \frac{\|\hat{\sigma} - \sigma_0\|_2}{\|\sigma_0\|_2} \tag{8}$$

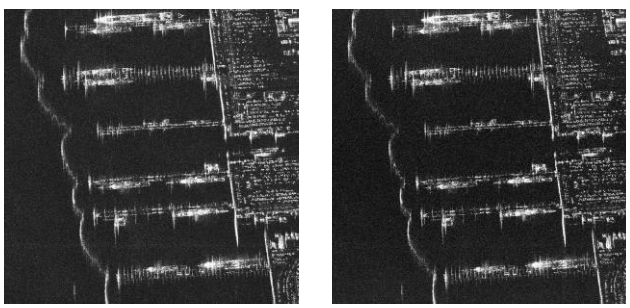
where $\hat{\sigma}$ denotes the recovered signal, σ denotes the original signal.



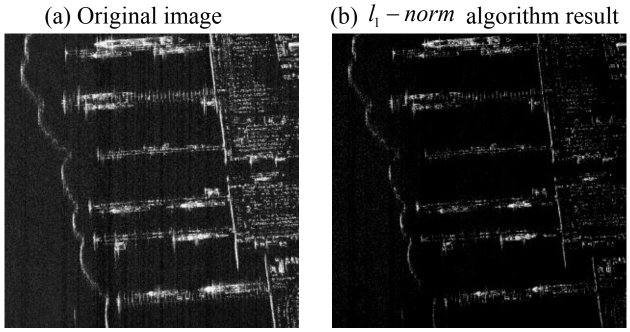
(a)No noise (b)SNR:10dB

Fig.2 Illustration of recovery error

Fig.2 shows the recovery error when sparsity is between 0% and 50% for l_1-norm, OMP, and BCS algorithms. The recovery error of l_1-norm algorithm is the smallest, followed by BCS algorithm. In the process of increasing the sparsity, OMP algorithm has the situation that the error increases suddenly. In the case of 10dB noise, the performance of l_1-norm algorithm is still optimal, while the performance of OMP algorithm becomes very poor.



(a) Original image (b) l_1-norm algorithm result



(c) BCS algorithm result (d) OMP algorithm result

Fig.3 Illustration of recovery result

Table.2 Evaluate parameters

	l_1-norm algorithm	BCS algorithm	OMP algorithm
Recovery error	0.0916	0.1125	0.2090
SSIM	0.5231	0.3071	0.0839

Echo simulation and reconstruction based on real SAR image without noise has also been done. The simulation parameters are the same as the last experiment. The human visual system mainly obtains the structural information from the image, so the image restoration quality can be evaluated through structural similarity (SSIM). It can be expressed as:

$$S(x,y)=\frac{2\mu_x\mu_y+C_1}{\mu_x^2+\mu_y^2+C_1}\cdot\frac{2\sigma_x\sigma_y+C_2}{\sigma_x^2+\sigma_y^2+C_2}\cdot\frac{\sigma_{xy}+C_3}{\sigma_x\sigma_y+C_3} \tag{9}$$

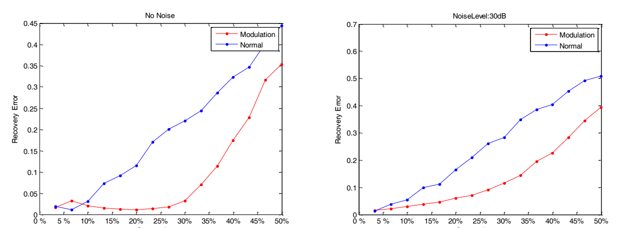
where x and y are the original image and the image to be evaluated, μ_x and σ_x represent the mean value and standard deviation, σ_{xy} denotes the covariance between x and y, C are constants to avoid having a 0 in the denominator. Fig.3 and evaluate indicators in Table.2 verify that the l_1-norm algorithm is optimal, and BCS algorithm comes next. OMP algorithm and above two still has gap.

4. PERFORMANCE ANALYSIS OF PSEUDO-RANDOM SPACE-TIME MODULATION

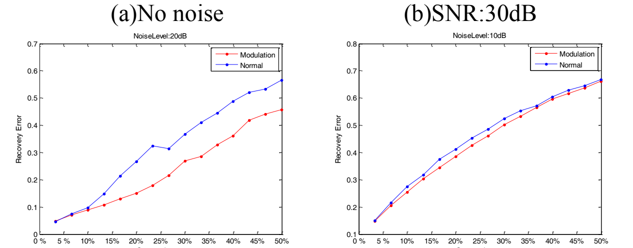
The following experiments adopt the parameters in Table.1, condition in section 3.2, and l_1-norm algorithm except for the phase settings.

4.1 Performance comparison of experiments with and without modulation

In this work, we compare the performance of the observation matrix without modulation and with uniformly distributed random phase modulation in $[0,2\pi]$. The simulations are carried out under the condition of no noise, 30dB, 20dB and 10dB SNR, respectively.



(a)No noise (b)SNR:30dB



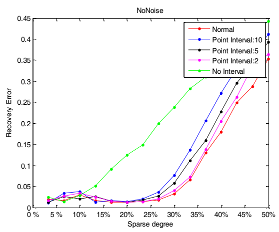
(c)SNR:20dB (d)SNR:10dB

Fig.4 Illustration of recovery error

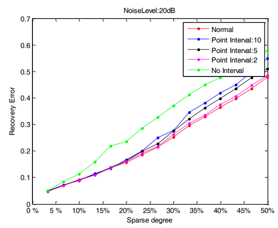
Fig.4 shows that the performance of the pseudo-random space-time modulation SAR is improved compared with traditional SAR in the presence of noise. When the sparsity is less than 20% and high SNR, the performance increases significantly. With the increase of the noise, the performance improvement with random phase modulation is more and more limited.

4.2 Performance comparison with different azimuth density of phase

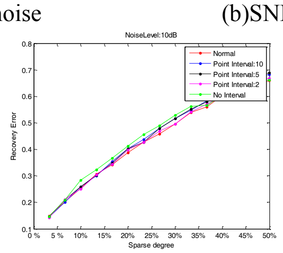
Ideally, the antenna should generate enough random phases in the azimuth direction to satisfy the observation requirements. Due to antenna technology, the number of random phases is limited. In this work, the random phase is generated at different densities along the azimuth direction, the remaining phases are obtained by adjacent phase interpolation. 2/5/10 were selected as the point target interval with phase for simulation.



(a)No noise



(b)SNR:20dB

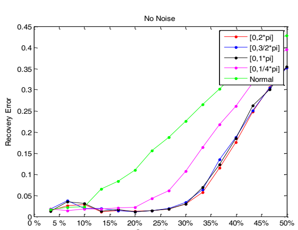


(c)SNR:10dB

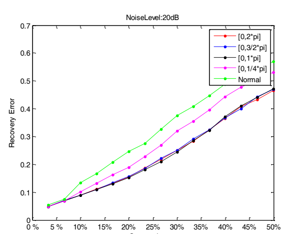
Fig.5 Illustration of recovery error

Fig.5 illustrates that with the decrease of phase density, the performance improvement obtained by modulation decreases continuously. When the sparsity is less than about 20%, there is little difference among the four. Noise also affects the performance of modulation. The smaller the noise, the more obvious the difference.

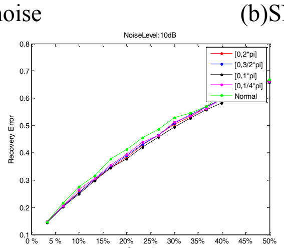
4.3 Performance comparison with different phase amplitude



(a)No noise



(b)SNR:20dB



(c)SNR:10dB

Fig.6 Illustration of recovery error

The amplitude of the modulation phase also affects performance. Fig.6 shows that the effect of modulation phase with amplitude variation in $[0,\pi]$, $[0,3/2\pi]$,and $[0,2\pi]$ is similar. The modulation performance of the phase varying in $[0,1/4\pi]$ is inferior to that of the other three situations. When the SNR decreases to 10dB, the difference between the four will be very subtle.

5. CONCLUSION

In this paper, we do a survey on the effects of noise, sparsity, phase density and phase amplitude on the pseudo-random space-time modulation technology performance. l_1-norm algorithm is the optimal algorithm in terms of recovery error. The matrix mutual coherence coefficients and the recovery error have proved the improvement of the modulation. When the SNR deteriorates to 10dB, the performance improvement due to modulation begin to be limited. With the decrease of phase density, the performance decreases continuously. The effect of modulation phase with amplitude variation range less than $[0,\pi]$ is similar. These experiments are very practical for designing a pseudo-random space-time modulation SAR system to observe different scenes.

6. REFERENCES

[1] Baraniuk R , Steeghs P . Compressive Radar Imaging[C]// Radar Conference, IEEE, 2007.

[2] Donoho D L . Compressed sensing[J]. IEEE Transactions on Information Theory, 2006, 52(4):1289-1306.

[3] Tello Alonso M , Lopez-Dekker P , Mallorqui J J . A Novel Strategy for Radar Imaging Based on Compressive Sensing[J]. IEEE Transactions on Geoscience and Remote Sensing, 2010, 48(12):4285-4295.

[4] Wenjiao Chen, Chunsheng, et al. Sub-Nyquist SAR Based on Pseudo-Random Time-Space Modulation.[J]. Sensors (Basel, Switzerland), 2018.

[5] Arjoune Y , Kaabouch N , Ghazi H E , et al. Compressive sensing: Performance comparison of sparse recovery algorithms[C]// 2017 IEEE 7th Annual Computing and Communication Workshop and Conference (CCWC). IEEE, 2017.

[6] Martinez-Lorenzo J A , Heredia Juesas J , Blackwell W . A Single-Transceiver Compressive Reflector Antenna for High-Sensing-Capacity Imaging[J]. IEEE Antennas & Wireless Propagation Letters, 2015:1-1.